EB-06 Beck Inertia Apparatus

Objective

To quantitatively describe the angular acceleration of a rotating object.

Theory

In this experiment we will study the angular acceleration due to the combination of an applied torque and a frictional torque. In the apparatus sketched in Figure One a wheel is free to rotate about a vertical axis. A string wrapped around the shaft and attached to mass M, which hangs from a pulley, produces a torque causing the wheel to speed up. Additionally, there will be a frictional torque τ_f due to imperfections in the bearings, which will tend to stop the wheel. Putting all the forces together, we can write Newton's equations for the system in the form

$$T = \mathbf{I}\,\alpha = r_s T - \tau_f$$

$$Ma = Mg - T$$

where **I** is the moment of inertia, T is the tension in the string, α is the angular acceleration, and the other quantities are shown in the Figure One.

The critical dimensions are r_s = 2.68 cm and the flag width L = 1.0 cm.

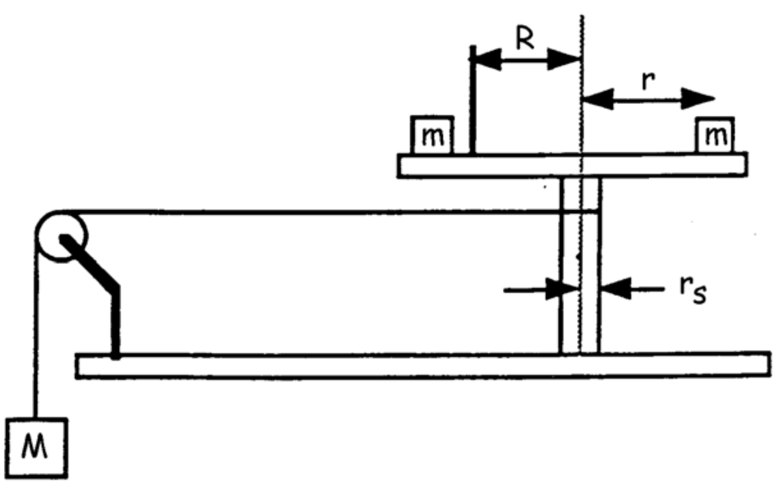

Figure One

Because the length of the string is fixed, the linear acceleration of M is related to the angular acceleration of the wheel by

$$A = r_s\,\alpha$$

These equations can be solved for the angular acceleration, yielding

$$\alpha = (M\,r_s\,g - \tau_f)/\,(\,\mathbf{I} + M\,r_s^2)$$

This expression is somewhat messy, but fortunately we can make the approximation that $M\,r_s^2$ is much smaller than **I** for our apparatus. With this simplification we can write

$$\alpha = (M\,r_s\,g - \tau_f)/\,\mathbf{I}$$

Assuming that τ_f is constant, we see that the wheel will have constant angular acceleration, a case which is familiar from your text.

Two situations are of interest here. In the first we allow the mass M to fall a known distance h, accelerating the wheel from rest to a final angular velocity ω_f. A little algebra on the constant acceleration equations gives

$$\omega_f^{\,2} = 2\,\alpha\,\theta_f$$

when the wheel rotates through the angle θ_f while the weight is falling a distance h. Using the relation $\theta_f = h/r_s$ between the total angular rotation and the distance M falls we obtain the more useful relation.

$$\omega_f^{\,2} = (2h/ r_s \mathbf{I}) (M r_s g - \tau_f)$$

From this result we see that a plot of $\omega_f^{\,2}$ vs M will be a straight line from which we can estimate both I and τ_f.

The other problem we set ourselves is to measure τ_f more directly by determining the time for the wheel to change its angular velocity by a known amount. The pertinent equation is

$$\omega_f = \omega_i + \alpha \; t_f$$

where ω_t is the initial angular velocity, ω_f is the final angular velocity and t_f is the elapsed time between the angular velocity measurements. Solving for α and then applying $\tau = \mathbf{I} \; \alpha$ we can get τ_f.

The moment of inertia of our apparatus is determined primarily by the mass and geometry of the wheel, shaft, and flag. We can increase the moment by adding up to three smaller masses at a distance r from the axis of rotation. If we call the moment of inertia of the fixed parts $\mathbf{I}_0$ then the total moment when all three masses are added is

$$\mathbf{I} = \mathbf{I}_0 + 3mr^2$$

This equation will provide a check on our results.

Experimental Procedure

Set up the apparatus so that M will fall into the sand box. Place the photogate so that the light beam intersects the axis of rotation, and can be broken by the flag once per rotation. Set the timer for “interval”. The angular velocity can be calculated from

$$\omega = L / R \,\Delta t$$

where L is the width of the flag, R is the distance from the flag to the axis and Δt is the timer reading. When you are ready to start the moment of inertia measurements, hook the loop of string over the peg on the shaft, and wind the string onto the shaft until M reaches a convenient height. You will want to start the shaft the same way each time, perhaps with the peg pointing at M. Reset the timer and release the shaft. When the string slips off the peg, immediately start the timer to determine Δt. You will also need to measure h, the distance M moves from the starting point to the point where the string comes loose.

Make measurements with M up to about 500 gm and make the appropriate plot. You should obtain two series of measurements, one for the wheel alone and another with weights added in such a way as to maximize **I**. Is the increase in moment of inertia that you deduce from the slopes consistent with what you estimate from the mass and position of the added weights? You should also estimate τ_f from this graph, and explain why this is not an accurate way to determine the frictional torque. You can get a better estimate of the friction by starting the wheel by hand and measuring the time required for it to slow down significantly . With the wheel spinning, arm the timer so it will measure the interval as the flag breaks the beam. This gives ω_f. Start timing with the stopwatch as the flag breaks the beam. After a reasonable interval, rearm the timer and then stop the watch when the flag breaks the beam again. This gives ω_f and τ_f. Use these numbers to find $\alpha$and hence τ_f using the values of I you determined.

Questions

In deriving the equations we use in the analysis, it was assumed that **I** is always much larger that Mr_s^2. Are your data consistent with this assumption? Is τ_f significant, relative to the torque you used to accelerate the wheel in the first exercise?