EB-07 Beck Torque Table

This experiment is a study in rotational equilibrium and center of gravity.

OBJECTIVES

Upon successful completion of this exercise you will have:

1. Verified the conditions for rotational equilibrium,

2. Calculated and experimentally verified the center of gravity for a system of objects, and

3. Experimentally determined the center of gravity of an irregular shaped object by two methods.

THEORY

Rotational Equilibrium: The tendency of a force to produce rotational effects on a body depends upon the magnitude of the force, its point of application on the body, and its direction. Specifically, this tendency is proportional to the **force moment** which is also called **torque, L**. A force produces a torque about an axis the magnitude of which is equal to the product of the force and the perpendicular distance between the line of action of the force (extended if necessary) and the axis. This perpendicular distance is called the moment arm of the force about the axis. The direction of the torque is assigned by the right hand rule, i.e. curl the fingers of the right hand in the direction of the tendency for rotation. The direction the thumb points is taken as the direction of the torque. For a body to be in rotational equilibrium, the sum of the torque tending to produce rotation in one direction about any arbitrary axis must be balanced by torque of equal sum tending to produce rotation in the opposite direction. That is, the sum of all torque about any axis must be zero. If the sum of all torque about an axis is not zero, then the body has angular acceleration about this axis and it is not in equilibrium. Rotational equilibrium about all axes is assured if rotational equilibrium about each of three mutually perpendicular axes is obtained. Thus, the condition for rotational equilibrium can be written as:

$$\sum(L_x)_i = 0$$

$$\sum(L_y)_i = 0$$

$$\sum(L_z)_i = 0 \qquad (1)$$

where $\sum(L_x)_i = 0$ means "the sum of the torque about the x axis" and the other notations have similar meanings with reference to the y and z axes. The balance table used in this experiment consists of a circular platform with rectangular coordinate markings mounted on a central hub. This hub is suspended on a pivot so that the center of gravity of the table and hub is lower than the point of suspension. The hub also carries a circular level for determining when the platform is horizontal. Masses can be added by placing them on the top of the table or by placing them on hangers suspended from holes in the table. If a point mass m, having weight w is placed on the balance table at position (x_i, y_i), as illustrated in Figure One, it will create a torque about the y axis of magnitude $w_i x_I = m_i\, g\, x_i$ and a torque about the x axis of magnitude $m_i\, g\, y_I$. By convention, the torque is assigned a + sign if it is in the +y or +x direction.

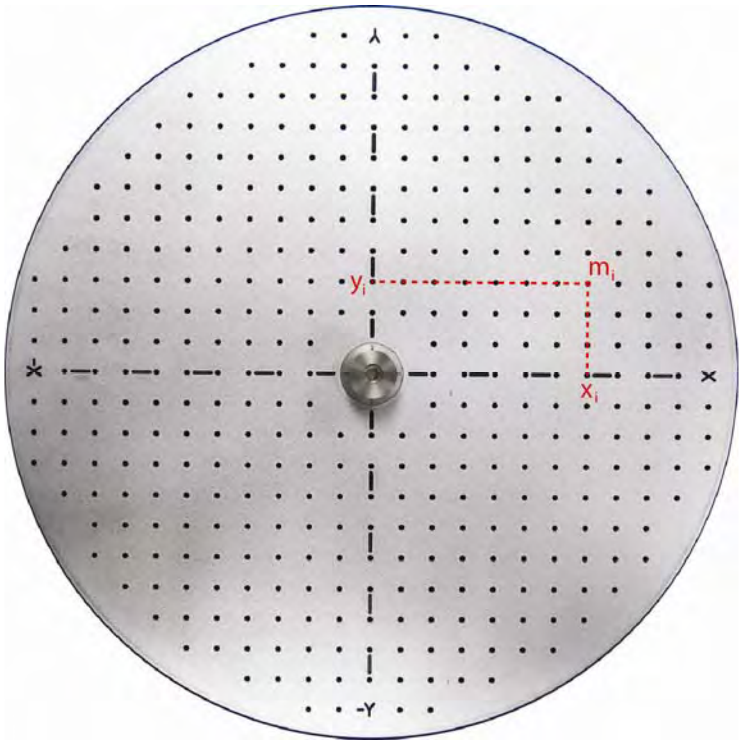

Figure 1

As sketched in fig.1, m would produce torque in the positive x and positive y directions. The total torque about the y axis, L , produced by several masses placed on the balance table would be

$$L = m_1 x_1 g + m_2 x_2 g + m_3 x_3 g + m_3 x_3 g \cdots + m_n x_n g$$

$$m_1 x_1 g + m_2 x_2 g + m_3 x_3 g + m_3 x_3 g \cdots + m_n x_n g = 0$$

or

$$m_1 x_1 + m_2 x_2 + m_3 x_3 + m_3 x_3 \cdots + m_n x_n = 0 \qquad (2)$$

Similarly, for the torque about the x axis,

$$m_1 x_1 + m_2 x_2 + m_3 x_3 + m_3 x_3 \cdots + m_n x_n = 0 \qquad (3)$$

Note that equations 2 and 3 are valid regardless of the units of mass and distance.

In developing equations 2 and 3, point masses, m_i, are assumed to be located at positions (x_i,y_i). In the introductory physics laboratory, slotted weights are used to load the balance table. Equations 2 and 3 are still valid for the slotted weights, provided the "center of gravity" of the weights are located at positions (x_i,y_i). The location of the center of gravity of the weights placed on top the platform may not be known exactly because of lack of symmetry. However, for the weights suspended from the hangers, the center of gravity will always be directly below the point from which the hanger is suspended.

Center of Gravity

A gravitational force acts upon each portion of an extended body. The total gravitational effect can be determined by summing the gravitational effects of all the individual portions of the body. However, if the gravitational field is uniform, a properly placed force equal to the total weight of the extended body may replace the individual weights. The point at which this single force must be applied is called the center of gravity of the body. Thus, the center of gravity is that point at which the entire weight of the object may be considered to be concentrated, and still give the same net torque as the extended object. The center of gravity so defined is a fixed point and its location is independent of the orientation of the body. The center of gravity of a system of several bodies can be easily computed if the centers of gravity of the individual bodies are known. If these several bodies are assumed to be in a rectangular coordinate system with the z axis vertical, the gravitational torque about the y axis of a weight w_i with its center of gravity located at (x_i, .y_i, z_i) is given by $w_i x_i$. The total gravitational torque about the y axis is then

$$L_y = m_1x_1 + m_2x_2 + m_3x_3 + m_3x_3 \cdots + m_nx_n \qquad (4)$$

This torque may also be represented as the product of the total weight of the bodies and the x coordinate of the center of gravity of the system of bodies. Thus,

$$L_y = (w_1 + w_2 + w_3 + w_4 \cdots + w_n)X \qquad (5)$$

From equations (4) and (5) it follows that the x coordinate of the center of gravity of the system is given by

$$X = \frac{w_1x_1 + w_2x_2 + w_3x_3 + w_3x_3 \cdots + w_nx_n}{w_1 + w_2 + w_3 + w_4 \cdots + w_n} \qquad (6)$$

Similarly

$$Y = \frac{w_1y_1 + w_2y_2 + w_3y_3 + w_3y_3 \cdots + w_ny_n}{w_1 + w_2 + w_3 + w_4 \cdots + w_n} \qquad (7)$$

If the system is then rotated until either the x axis or the y axis is vertical, it can also be shown that

$$Z = \frac{w_1 z_1 + w_2 z_2 + w_3 z_3 + w_3 z_3 \cdots + w_n z_n}{w_1 + w_2 + w_3 + w_4 \cdots + w_n} \quad (8)$$

An extended body suspended from a string is free to rotate about any horizontal axis passing through the point of suspension. When such a body comes to rest, the gravitational forces must produce no net torque about any horizontal axis through the point of suspension. One way of locating the center of gravity of an irregular shaped extended body is to suspend it from a string and allow it to come to rest. Then the center of gravity will lie on a line, which is an extension of the supporting string. This may be done for several points of attachment of the string and the extended lines will all intersect one another at the center of gravity of the body. It follows that a body suspended from its center of gravity will have no tendency to rotate about this point. This is regardless of its orientation or if suspended from any other, it will assume a final position where the center of gravity is directly below the point of suspension.

PROCEDURE

The EB-07 Torque Table is supplied with five weight hangers but does not include masses for the experiment. The actual masses used are not important as long as their values are known. The same principles can be verified with different mass values. Likewise, the irregular mass used in the last part is not supplied. Since there are no requirements on it beyond being flat, it can easily be fabricated.

A. Masses Placed on Top of Table

1. To estimate the approximate center of gravity of a slotted weight, balance it on a fingertip and note the approximate location of the balance point. Assume the uncertainty of the position of the center of gravity of the slotted weight to be about 1/2 the thickness of your fingertip and record this number on the data sheet.

Note that the holes on the balance table are drilled on 2.5 cm centers and that one could only estimate the position of an object located between the drilled holes to within about 4mm. Thus, if the assumed center of gravity of a slotted weight placed on top of the table coincides with a drilled hole, the uncertainty in the position of the weight would be only the uncertainty in the c.g. of the weight. However, if the assumed c.g. lies between the drilled holes, the uncertainty in its position would be the uncertainty in the c.g. and the 4 mm scale uncertainty. Note also that the holes nearest the center of the table are 5 cm from the center. Perform an initial balance of the table placing a small mass in such a position as to center the bubble in the level.

2. Place a 100g mass at (0, _) and balance with a 50g mass placed on the y axis. Compute the torque produced by each of these masses. When

calculating the uncertainty in torque, assume no uncertainty in the masses. Values shown in parentheses are the x and y coordinates of the masses, with the center of the table at (0,0).

3. Without disturbing the masses in Step 2 above, add another 100g mass at (_,0) and rebalance by adding a 50g mass along the x axis. Compute the gravitational torque produced by each of these added masses. Calculate the total torque about the +x and +y axes, and the -x and -y axes. Is the total torque about the positive axes equal to the total torque about the negative axes, within experimental error?

4. Without disturbing the 100g masses, find different off axis positions for each of the two 50g masses that will also produce a balance. Compute the torque about the x axis and about the y axis produced by the four masses. Is the total torque about the positive and negative axes equal, within experimental error?

5. Without disturbing the 100g masses, remove the two 50g masses and replace with a single 100g mass placed in the position where it balances the other two 100g masses. Compute the torque about the x axis and about the y axis. Is the torque about the positive axes equal to the torque about the negative axes, within experimental error?

B. Masses Suspended From Hangers

1. When the masses are suspended from hangers beneath the table, there is no uncertainty in the position of the c.g. of the masses. There may be a small uncertainty in the masses needed for balance.

   Perform an initial balance as in procedure A-1 with buttons in all holes from which hangers are to be suspended in parts B-2, 3, and 4. When the buttons are repositioned as for the hanger positions of part B-5, the initial balance must be repeated with the buttons in the new position.

2. Place a total mass (including mass of the hanger) of 130g and balance by adding the required mass at (0, __). Compute the torque due to each of the suspended masses.

3. Add a second mass of 70g total at (_,0). Balance by adding the necessary hanger and masses at (_, 0). Compute the torque due to each of the added

masses. Is the total positive and negative torque equal?

4. Leave the 70g and 130g masses in the same position as for part 3 and remove the other two weights (including hangers). Place a 200g mass on top of the platform in a position to produce balance. Is the total positive and negative torque equal, within experimental error?

5. Place hanger buttons at (0, _), (_, 0), and (-7 , -8) and make an initial balance. Suspend 200g (total) at (-7, -8) and balance by suspending hangers and masses from the other two buttons. Is the total positive and negative torque equal?

6. Suspend masses of 100g, 150g, and 200g (include hangers) from the three buttons positioned for part 5. Use a single 200g mass on top of the platform to balance it. Is the total positive and negative torque equal, within experimental error?

Leave the masses undisturbed for the next part.

Center of Gravity of a System of Masses

1. Use the data in part B-6 and equations 6 and 7 to calculate the c.g. of the 100g, 150g and 200g masses. Show your calculations on the data sheet.

2. Leave the 200g mass undisturbed on the top of the table. Remove the 100g, 150g and 200g masses and rebalance the table with a 450g mass on top of the table. Record the coordinates of the 450g mass as the measured c.g. of the 100g, 150g and 200g masses. Are the measured and calculated coordinates of the c.g. equal, within experimental error?

   If not, calculate the percent discrepancy in each of the coordinates.

Center of Gravity of an Irregular Object

1. Obtain a piece of adhesive paper and an irregular shaped piece of metal from the center table. Trace the shape of the metal on the paper and cut out this shape. Peel the backing from the adhesive paper and attach the paper to the piece of metal. Determine the mass of the metal and attached paper with a laboratory balance. Record the mass.

2. Remove all buttons from the balance table and make an initial balance. Place the metal piece on

the top of the balance table near the center, papered side up. Balance the table with two 100g masses placed on top of the table, one on the x axis and the other on the y axis. Record the positions of the two masses, and the uncertainties in position. Calculate the total torque produced by the two 100g masses about the x and y axes. Leave the masses undisturbed until you have completed Part 3 below.

3. From equation 5 it is noted that the total torque produced by the metal about the y axis is the product of its weight and the x coordinate of its center of gravity. Since the table is balanced, this torque must be equal to the total torque produced by the two 100g masses about the y axis. By equating these torques one can calculate the x coordinate of the center of gravity of the irregular piece of metal. Do this calculation and draw a line on the paper on the metal at the calculated x coordinate of the center of gravity. Make a similar calculation for the y coordinate and draw a line on the paper for this coordinate. Show your calculations on the data sheet. The point at which the two lines cross is the calculated center of gravity of the piece. Mark this point "calculated c.g." on the paper.

4. Remove the metal piece from the balance table and suspend it by a small nail through one of the holes drilled in the metal. Since the c.g. of the metal lies directly below the point of suspension, use a plumb line to locate all points below the suspension. Draw a line on the paper along the plumb line. Repeat this process for another drill hole. The point where the two plumb lines cross is the measured c.g. of the metal. Mark this point "measured c.g." on the paper. Peel the paper from the metal and attach it to the data sheet in the space marked "D-4".

DATA and ANALYSIS

A. Masses on Top of the Table

1. Uncertainty in c.g. of slotted weights cm

   Uncertainty in Balance Table position 0.4 cm

Table for steps 2 and 3:

Mass (g)	Position (cm)		Torque (g*cm)		Torque (g*cm)		Position (cm)		Mass (g)
	x	y	about x	about y	about x	about y	x	y	
100	0			0		0	0		50
100		0	0		0			0	50
Total torque, 100g masses							Total torque, 50g masses		

Step 4:

									50
									50
Total torque, 100g masses							Total torque, 50g masses		

Step 5:

Total torque, 100g masses									100

B. Masses Suspended from Hangers

Table for steps 2 and 3:

Mass (g)	Position (cm)		Torque (g*cm)		Torque (g*cm)		Position (cm)		Mass (g)
	x	**y**	**about x**	**about y**	**about x**	**about y**	**x**	**y**	
130	0						0		
70		0						0	
		Total torque					Total torque		

Step 4:

		Total torque							200

Step 5:

200								0	
							0		
		Total torque							

Step 6:

100	0								200
150		0							
200									
		Total torque							

C. Center of Gravity of a System of Masses

1. Show Calculations

   Calculated x cm

   Calculated y cm

2. Measured x cm % discrepancy x cm

   Measured y cm % discrepancy y cm

D. Center of Gravity of an Irregular Object

1. Mass of the Object gm

Table for Step 2:

Mass (g)	Position (cm)		Torque (g*cm)	
	x	y	about x	about y
100	0			0
100		0	0	
Total torque, 100g masses				