EB-04 Beck Centripetal Force Apparatus

Introduction

The Beck Centripetal Force Unit (EB-04), is a simple apparatus designed for experiments in uniform circular motion. Not only can the forces be visualized and recognized— they can actually be produced and measured directly. Many students welcome the chance to do a dynamics experiment with good mechanical equipment.

Design features of the original unit of 1972 are still evident. A minor improvement is the use of chrome-molybdenum steel for the lower part of the center rotating shaft to increase rigidity. For long-time good appearance, we have always used stainless steel for parts that are handled. Although stainless is expensive and not particularly cooperative in the machine shop, it does make beautiful equipment for many years of use.

Assembly

Please read this section carefully before unpacking and assembling the machine. Please do not dispose of any packing materials until you have assembled the unit and are positive that it is complete.

Hang the test mass from the cord, being sure that it does not bump a radius point. Make small adjustments in the height of the hanging mass by turning the eyebolt in its top. The fitting on the mass should be on the same level as the spring attaching screw. Finally, slide the rod in the vertical shaft so that the mass, hanging directly above the radius point of your choice. Put the spring in place and check the rotation. It should be true and smooth. Adjust the position of the counterweight to balance the assembly.

Design Features

The apparatus is a rotating assembly having a suspended test mass so that the centripetal force acting on it is produced by a spring as the mass rotates at a constant speed. Provision is made for determination of the radius of rotation and the measurement of the force involved. This apparatus is operated by hand.

The vertical shaft has a 1 1/4" diameter and a height of approximately 8 1/4". For easy rotation, the top section of this shaft is knurled. A horizontal rod fits through the shaft below the knurled section. The test mass hangs from a short crossbar near the end of this rod. A threaded hole near the lower end of the shaft carries a screw that adjusts the attachment point for the inboard end of the spring. The counterweight on the other end of the horizontal rod provides static balance. A safety cap on the aft end of the rod prevents the counterweight from flying off. The whole assembly rotates in two large ball bearings.

The test mass has a pointed lower end for easy determination of its radius position while rotating. An eyebolt serves to hang it from a loop of heavy cord. Adjust the height by turning the eye bolt in the test mass and lock it with the large knurled nut. Two fittings in the sides of the mass serve to attach the mass to the stainless steel spring on one side and a line to a weight hanger on the other.

The radius indicator measures the radius of rotation and is capable of tracking movement in the 14 to 22 cm range. The indicator also supports a ball bearing pulley for static deflection of the bob.

Use, Maintenance, and Repair

As with all student labs, it is important that those conducting the use of the centripetal force unit become thoroughly familiar with its operation and use.

Although our products are known for their durability, it is a very good idea to frequently inspect the centripetal force unit. A quick pre-lab check may be good, especially if you make sure that you are seen doing it. Chances are that the unit will get better care when all know that it has just been examined.

The post-lab inspection is valuable because you can make any minor adjustments or repairs before the equipment's next use. The unit is very dependable and trouble free, but cords wear and small parts sometimes migrate to the floor or into a pocket. Knowing this, we try to use standard hardware items that can be found in the corner hardware store. If missing parts are not available locally, please contact us for any replacements you may need.

Keep the unit clean. We suggest a liquid detergent on a soft cloth to remove soil and markings. Carefully rinse and wipe dry. Avoid getting any water down into the bearings.

Although that small spring mounting screw is stainless, the material of most of the shaft at the point where the screw is located is chrome-molybdenum steel, so it is a good idea to periodically lubricate the threads of the screw with oil and turn it back and forth in the shaft. We suggest that this maintenance be performed once a year.

UNIFORM CIRCULAR MOTION
by

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May 1988

Dear Fellow Teacher,

I've been a teacher of Physics at the University of Texas at Arlington for some 22 years and have been continually involved with the student laboratories that accompany our several introductory courses. As a result of some of that work I was able to make a small contribution to the design of Mr. Beck's apparatus to study uniform circular motion. I was flattered when he asked me to write a set of exercises to accompany the equipment. As I neared completion of that work, I asked him for the opportunity to share some of my thoughts concerning laboratory instruction with you. That is the purpose of this letter.

The Beck UCM equipment is special and I think you need to give it special consideration. You may not have had your students doing any lab work with UCM. They surely haven't been doing 7 to 10 hours laboratory work in this area. Yet, the Beck UCM equipment allows your students to profitably invest at least this much time with the topic of uniform circular motion. Is it worth it? Teachers will ultimately have to answer this question for themselves, but it is here that I would like to share some of my thoughts with you.

It's not that UCM as a topic is worth so much of the students time, but your students will benefit from extensive use of this equipment for several reasons.

(1) The equipment looks good! It really looks good. It looks like it was made with a purpose. Students appreciate exposure to good equipment. If all your lab equipment looked as good as this, your students' attitude toward science in general and lab in particular would improve significantly.

(2) It works well! The careful student can achieve great results with this equipment. It is frustrating to the good student to work hard and still get poor results because the equipment isn't worthy of the student's effort. Still, the sloppy student will get sloppy results. There may be a real object lesson here. Before, everyone got poor results because the equipment was poor. Now there can be some reward for careful scientific work.

(3) The equipment enables the student to become involved in what I call the process of science. There are data to be gathered, graphs to make, slopes of curves to determine (with emphasis on units) and comparisons with theoretical expectations to be made. These are areas in which college freshmen are so weak!

It's this last point that I think is so important! In my opinion, it would be far better to do only a few things in lab and do them really well than to do many things superficially. If you agree with this fundamental philosophical position, the Beck UCM equipment will enable you to do a few things very well with your students.

Sincerely,

Glen Edward Terrell

Associate Professor of Physics

Uniform Circular Motion

Background

The serious physics student should give special attention to the study of uniform circular motion, or UCM for short. Special attention to UCM is justified because the motion of so many objects is accurately (or at least approximately) described as UCM: from children on a merry-go-round, to charged particles in a magnetic field, to our own motion around the Sun.

Uniform circular motion is commonplace, but not simple. In fact, it is the most complicated motion considered in an introductory course.

Recall Newton's second law of motion,

$$\sum \mathbf{F} = ma \quad (1)$$

which says that the sum of all the forces on an object of mass m equals the product of the mass of the object and the acceleration of the object. Recall also that the acceleration and the force are parallel vectors.

What is described as uniform circular motion occurs when the net external force acting on an object satisfies the following three conditions:

1. It has a constant magnitude.
2. It continually acts on the object at right angles to the velocity vector of the object.
3. It is continually directed toward the same point in space.

When these conditions are satisfied by the net external force acting on the object, the object moves in a circle (the center of this circle is the point toward which the net external force points) with constant speed. Such a force is sometimes called a centripetal (or center-seeking) force. All this is depicted in Figure 1.

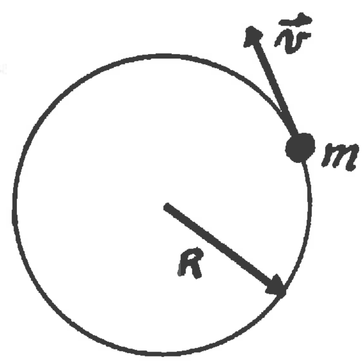


Figure 1

For this type of motion, the magnitude of the acceleration of the object is related to the speed of the object, v, and the radius of the circle, R, on which it moves by the equation:

$$a = v^2 / R \quad (2)$$

Therefore, since F = ma (if no other external forces act on the object),

$$F = m v^2 / R \quad (3)$$

Equation [3] is essentially a theoretical result. The object of this set of exercises is to test the validity of this theoretical result.

A Preliminary Consideration

It is difficult (the equipment is expensive) to measure the speed of an object directly. However, the speed can be determined indirectly from $v = d/t$ where $d = 2 \pi R$, the distance around the circle and $t = T$ where T is the period of the motion or the time required for the object to go around the circle one time. With this one modification of thought, equation (3) can be rewritten as:

$$F = 4 \pi^2 m R / T^2 \quad (4)$$

The value of the parameters m, R and T can be measured directly and the force F can be measured indirectly. Equation (4) is the theoretical expression that will actually be examined.

More of the Physics

Refer to the picture of the equipment or, if possible, examine the equipment itself. When the shaft is spinning and the mass (sometimes referred to as the bob, like in pendulum bob) is moving in a circle, the mass is subject to three forces. One force is its weight directed downward (as always), the second force is due to the string harness from which the bob is suspended, and the third force is due to the spring. The direction of the force exerted on the bob by the string harness is generally upward and in the plane of the strings. The direction of the force exerted on the bob by the spring is in the direction of the extension of the spring.

It is important that while measurements are being made, the plane of the strings be vertical so that the force exerted on the bob by the strings is vertical. It is equally important that the spring be horizontal so that the force exerted on the bob by the spring, is horizontal.

When the forces are aligned in this way, the force exerted on the bob by the strings and the weight of the bob exactly counterbalance each other and the net external force acting on the bob is just that due to the spring!

Therefore, for each measurement be sure that the strings are vertical and that the spring is horizontal.

Measurement Technique

The Centripetal Force Apparatus comes with a weight hanger but does not include masses for the experiment. The values used in the following description are typical of those used in a major university. The actual masses used are not important as long as their value is known. The same principles can be verified with different mass values.

For each measurement do, or at least consider, the following series of steps.

1. Disconnect the spring from the bob and let the bob hang straight down.

2. Establish the radius of the circle on which the bob is to move by loosening the screw in the horizontal arm and moving the arm until the bob hangs directly over

the proper centimeter mark on the base of the apparatus. Tighten the screw so that the arm will not slip when the shaft is spun.

3. Attach the spring to the bob. Now attach the horizontal string to the bob, pass the string over the pulley, suspend a mass hanger on this string and put just enough mass on the hanger (call it M_h) so that the spring is lengthened just enough to pull the bob back over the same proper centimeter mark. When aligned in this way, the forces due to the weight and vertical strings exactly negate one another, and the magnitude of the force exerted on the bob by the spring is equal to the force exerted on the bob by the horizontal string, which is itself equal to the weight of the mass on the hanger, or finally,

$$\Sigma F - F_{cent} = (M_h) g . \qquad (5)$$

4. Remove the mass hanger from the horizontal string and get the horizontal string out of the way.

5. Spin the shaft faster and faster until the bob is moving in the circle of the correct radius. When this is done correctly the bob will, during each revolution, pass precisely over the correct centimeter mark selected in step 2 above. This step is very important if you are to obtain good results! Practice spinning the shaft keeping the bob in a constant radius circle for 30 or 40 turns.

6. To measure the period T, one student should get the bob going at the right speed in the right circle, start a count down of the revolutions (-4, -3, -2, -1, 0, 1, 2,...), and a second student should start a clock on the count of 0. The student spinning the shaft should maintain UCM of the bob at the correct radius while counting the turns aloud. The second student should stop the clock at the count of 30 (or some other predetermined count). Record the total time and divide by 30 (or the number of turns) to get T.

Viewed in a most basic fashion, equation [4] represents a mathematical relationship between four parameters: F, m . R and T. How should one proceed experimentally to verify (or contradict) such a relationship? The thoughtful consideration of a question like this is of great value to the laboratory student. Although there may be many answers to this question, perhaps the best response is "keep two of the four parameters fixed, vary the value of a third one and measure the resulting value of the fourth one." This is the principle consideration in the design of the experimental procedure for the exercises that follow and should become a central consideration in future experimental procedures of your own design.

Exercise 1

Following the important forethought on the previous page, if F and R are held fixed, or constant, what will be the mathematical relationship between the other variables T and m? Rewriting [4] yields,

$$T^2 = (4\pi^2 R / F) m \quad (6)$$

Thoughtful consideration of equation 6 should lead the maturing experimentalist to the following conclusion: since R and F are being held constant, one can vary m and measure the resulting value of T. Further, since the contents of the parentheses are constant, a plot of T^2 versus m should result in a straight line. Still further, the y intercept of this straight line should be 0 and the slope of the line should be $(4\pi^2 R / F)$. The conclusion forms the pattern for the experimental design that follows.

1. Measure T for various values of m keeping R and F constant.
2. Compute values for T^2 for each value of m.
3. Plot T^2 versus m and check for a straight line.
4. Draw the best straight line possible through the origin, determine the slope of the line and compare it with the theoretical expectation.

Procedure

1. Disconnect the spring from the bob. Remove the bob including the screw and nut on top of it. (The nut and screw are to be considered as a part of the bob, which makes sense since they move with the bob, and the mass of each contributes to the mass of the object undergoing UCM, i.e., the bob.) Determine the mass of the bob and reconnect it to the apparatus. Record the mass on the data sheet.

2. Position the horizontal arm so that the radius of the circle on which the bob will move is R = 18 cm. (Actually any radius near 18 cm will do.) Record the value of R on the data sheet.

3. Reconnect the spring to the bob. Adjust the position of the horizontal screw to which the spring is attached until the spring exerts a force near 7 N (700 gm weights) on the bob when it hangs directly over the 18 cm mark. Record this measured value of F on the data sheet. The same value of F will be maintained for all the measurements in this first exercise.

4. Measure the time for N (perhaps 30) revolutions. Record the value of N and the time for N revolutions in the data table.

5. Compute and record the value of the period T.

6. Add 50 grams mass to the bob by putting a slotted 50 gram mass under the nut that is on top of the bob. Be sure that the nut is secure. Repeat steps (4) and (5).

7. Continue increasing the mass of the bob in 50 gram steps until a total of 300 grams have been added or you reach the limit of the equipment, whichever comes first. Just how many masses you can add depends on the thickness of the slotted masses available. If you have a choice, use the thinner slotted 50 gm masses.

8. Repeat steps (4) and (5) for each added mass.

Analysis

1. For each measurement, compute values of T^2 and record the results in the data table.

2. Make a graph of T^2 versus m on regular graph paper. Values of T^2 should be on the y-axis and m should be on the x-axis.

3. This is a crucial step! Using a straightedge (a clear straightedge is preferred) draw the best straight line possible through the data that goes through the origin. You can obtain a more exact best-fit line by using the least-squares method in any scientific calculator, which will also provide the slope.

4. The straight line now represents your determination of the relationship between T^2 and m. The data points just lead you to this conclusion. Now ignore the data points and pay attention to the line only since it represents your "scientific truth" in this case. Select two points on the line (not data points) that are far apart. The farther apart these two points are the better. Use these two points to determine the slope of the line. Do this mathematical work on the graph paper with the graph. Be careful of the units! The slope does have units. Record this experimental value of the slope on the data sheet.

5. According to the theory, (see equation (6)) the slope of the line should be:

$$\text{slope} = 4\,\pi^2\,R\,/\,F \qquad (7)$$

Using the (constant) values of R and F for this series of measurements, compute the theoretical value of the slope. Record this value on the data sheet also.

6. If the theory is correct and your experimental work was carefully performed, these two values of the slope should agree. So compute the percent difference between these values using the following equation.

$$\text{percent diff} = 100 \{ slope_{exp} - slope_{theo} \} / slope_{exp} \quad (8)$$

The closer the agreement the better. If the agreement is poor, you should perhaps examine what you have done to be sure that you haven't made a mistake somewhere along the way. Remember, if what you believe is correct and your experimental work is correct these two values MUST agree.

Data Sheet
Exercise 1

Procedure

1. Value of the constant radius, R m.
2. Value of the constant of force, F = N. ($F = M_h g$)
3. Mass of bob, m = kg.

DATA TABLE I

added mass kg	total mass of bob kg	N	time for N revolutions (sec)	T (sec)	T^2 (sec^2)
0					
0.050					
0.100					
0.150					
0.200					
0.250					
0.300					

Analysis

Measured slope of the straight line =

Theoretical slope of the straight line =

Difference in the two values = %

Exercise 2

Now suppose that the mass of the bob and the net external force acting on it were both kept constant. How would the period of the motion change as the radius of the motion, R, was changed? Looking back at equation [4] and solving it for T in terms of the other variables yields

$$T^2 = (4\pi^2 m / F) R \qquad (8)$$

Thinking of this equation in the same way we did equation [6], one can conclude that if a series of measurements are made for which m and F are both constant, then the contents of the parentheses are constant. Therefore a plot of T^2 versus R should be a straight line with a y intercept equal to zero and a slope equal to ($4\pi^2$ m / F). Therefore the experimental design will be as follows:

1. Measure T for various values of R without changing m or F.
2. Compute values of T^2 for each value of R.
3. Plot values of T^2 versus R and check for a straight line.
4. Draw the best straight line possible through the origin, determine the slope of the line and compare it with the theoretical expectation.

Procedure

1. Disconnect the spring from the bob. Remove the bob from the apparatus and determine the mass of it. Record the mass of the bob on the data sheet.

2. The spring attaches to a screw in the central vertical shaft. Move this screw into the shaft as far as possible in order to get the spring as close to the central shaft as possible. Suspend the bob from the string harness and position the horizontal bar so that the bob hangs directly over the 14 cm mark. Now attach the spring and the horizontal string to the bob. Pass the string over the pulley, put a mass hanger on the string and put a total of 700 grams on the string. Now note the position of the bob. The idea is to get the bob directly over the 14 cm mark with the screw as short as possible.

   If the spring has pulled the bob closer to the shaft than the 14 cm mark, you need to disconnect the spring from the bob and loosen the screw or add 50 grams to the mass hanger.

   If the 700 gram weight load on the string pulls the bob out beyond the 14 cm mark, remove 50 grams from the hanger. If necessary, remove a second 50 grams and so on. Finally, adjust the position of the screw until the bob is directly over the 14 cm mark.

The adjustment described in this section is tedious but do it carefully. The desired result is this: with the spring and the string both attached to the bob, the bob should be in equilibrium directly over the 14 cm mark and the screw should still have at least 4 cm outward movement remaining.

When you think you have the adjustment just right, go through it once more. Disconnect both the spring and horizontal string. Be sure that the bob hangs precisely over the 14 cm mark. Reattach the string and spring and make any needed adjustment in the screw to which the spring is attached.

Record the total mass load on the hanger, M_h, the weight of which is equal to the net external force.

Compute and record the value of the constant force (= M_h g)

3. Remove the horizontal string, or at least get it out of the way. Get the bob rotating with constant speed at the correct radius and, using the same technique as before, measure the time required for N revolutions. Record both these values in the data table. Compute and record the time for one revolution, T.

4. Now to change R without changing F. Disconnect the spring from the bob. Reposition the horizontal bar so that the radius is increased by 1 cm. The screw to which the spring is attached has 28 threads per inch. That translates to 11 threads per centimeter. So loosen the screw 11 turns (22 half turns may be easier to count). Do this carefully. If you miscount, it is impossible to get back to where you started with certainty. Reattach the spring to the bob. Put the horizontal string back on the bob with the same mass load to check that the bob is indeed in equilibrium just over the R_{min} + 1 cm mark and will experience the same centripetal force as before the radius was increased.

   Measure and record the time for N revolutions. Record the value of N. Compute and record the value of T.

5. Repeat step (4) until you reach the limit of the equipment.

Analysis

1. Compute and record values of T^2 for each value of R.

2. Make a graph of T^2 versus R with R on the x axis.

3. Using the same technique used in the first exercise, determine the value of the slope of the best-fit line. Again, be careful of the units! Record this value of the slope on the data sheet.

4. According to the theory, (see equation [8]) the slope should be:

$$slope_{theo} = (4 \pi^2 m/F).$$

   Use values of the constant mass and force to compute the theoretically expected value of the slope. Record this value on the data sheet.

5. Again, if what we believe in our head (or is it the heart?) is correct and what we do in the laboratory is valid these two values MUST be equal. Check this by computing the percent difference between the two values.

Data Sheet

Exercise 2

Procedure

1. Value of the constant mass, m = kg.
2. Mass on hanger corresponding to F_{cent}, M_h = kg.
3. Value of the constant force, F = N. ($F = M_h g$)

DATA TABLE II

increase in R above minimum (m)	R (m)	N	time for N revolutions (sec)	T (sec)	T^2 (sec^2)
0	0.14				
0.010					
0.020					
0.030					
0.040					
0.050					
0.060					

Analysis

Measured slope of the straight line =

Theoretical slope of the straight line =

Difference in the two values of the slope = %

Exercise 3

Now suppose that the same body (constant mass) experienced UCM in a constant circle. How would the period of the motion, T, change if the net external force acting on the bob F changes? Looking back at equation [4] and solving it for T (actually T^2) in terms of the other variables yields:

$$T^2 = (4\pi^2 m R)(1/F) \qquad (9)$$

Thinking of this equation in the same way we did equation [6], one concludes that if a series of measurements were made for which both m and R were constant, then the contents of the first parentheses would be constant and a graph of T^2 versus (1/F) should be a straight line. Further, the y intercept of the line should be zero and the slope of the line should be ($4\pi^2$ m R). Therefore, the experimental design will be as follows (are you getting the big picture yet?).

(1) Make measurements of T for various values of F without changing either m or R.

(2) Compute values for T^2 for each value of F. Compute values of (1/F) for each value of F.

(3) Plot T^2 versus (1/F) with T^2 on the y-axis and check for a straight line.

(4) Draw the best-fit line through the data and the origin and compare the slope of it with the theoretically expected value.

Procedure

1. Remove the bob from the apparatus and determine its mass. Record this value on the data sheet.

2. Suspend the bob again from the string harness and position the horizontal bar so that the bob hangs over the 18 cm mark.

3. Attach the horizontal string and the spring to the bob. Put a total of 700 grams on the string and adjust the position of the screw until the bob hangs in equilibrium just above the 18 cm mark.

4. Remove the string from the bob, get the bob rotating at constant speed in a circle over the 18 cm mark and measure the time for N revolutions. Record both N and the time in the data table. Compute and record the value of T.

5. Reattach the string to the bob and increase the mass on it to 400 grams (an increase of 100 grams). Adjust the position of the screw until the bob again hangs in equilibrium over the proper

centimeter mark. Again measure the time required for N revolutions. Record the value of N and the time required. Compute and record the value of T.

6. Continue to increase the net external force by an amount equal to the weight of 100 grams repeating the measurements and recordings in step (5) until five measurements have been made or you reach the limit of the equipment, whichever comes first.

Analysis

1. For each measurement compute and record F (=Mug), 1/F and T^2.

2. Make a graph of T^2 versus (1/F) with T^2 on the y-axis.

3. Draw the best-fit line through the origin and the data. Compute the value of the slope of the line and record the results on the data sheet.

4. According to the theory (see equation (9)), the slope of the line should be:

$$slope_{theo} = 4\pi^2 m\ R \qquad (10)$$

Use the constant values of m and R to compute the theoretically expected value of the slope. Record the results on the data sheet.

5. Compare the two values of the slope by computing the percent difference between them. Record the results on the data sheet.

Data Sheet

Exercise 3

Procedure

1. Value of the constant mass, m = kg.
2. Value of the constant radius, R = m.

DATA TABLE III

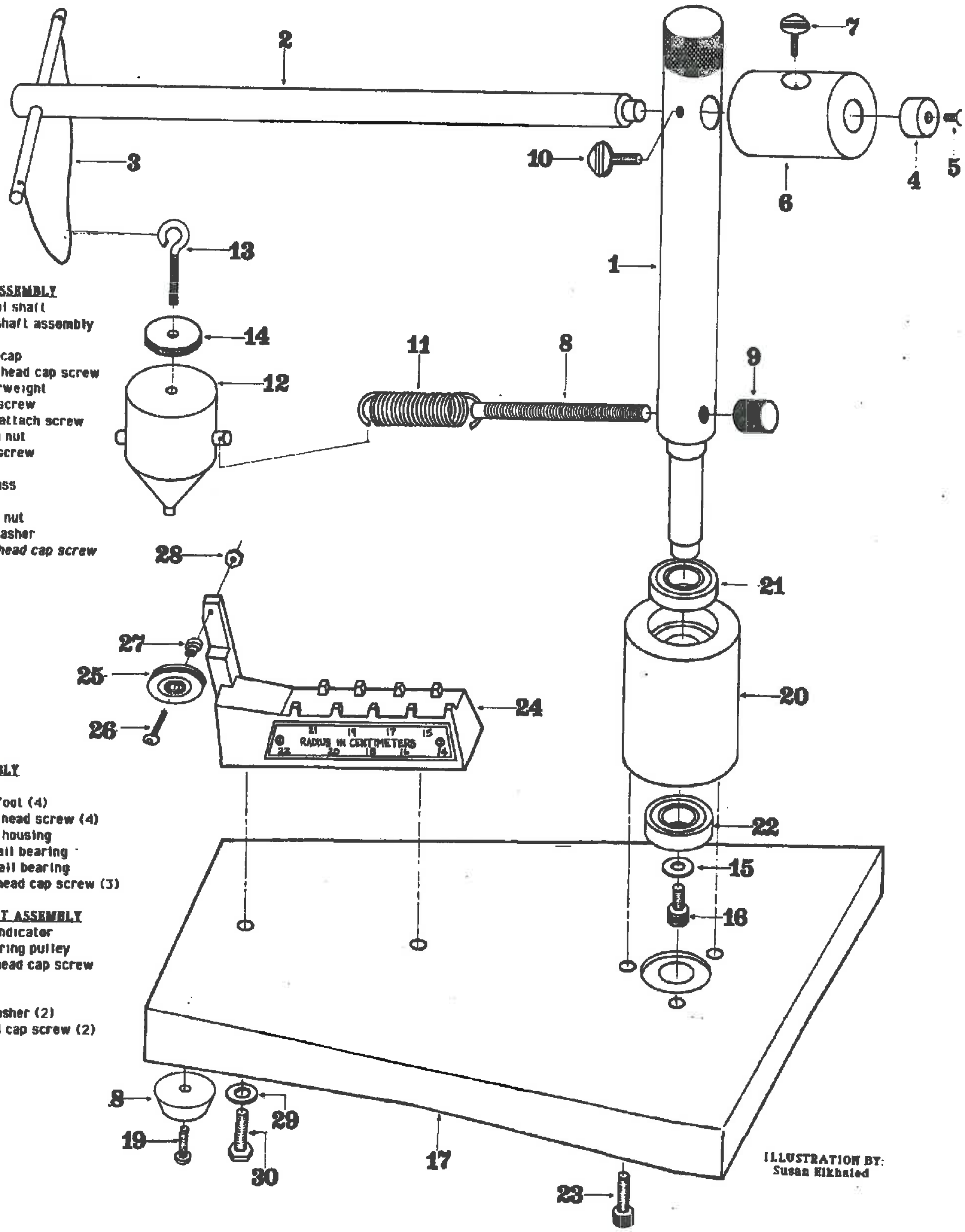
M_h (kg)	F (M_h g) N	(1/F) N^{-1})	N	time for N revolutions (sec)	T (sec)	T^2 (sec^2)

Analysis

Measured slope of the straight line =

Theoretical slope of the straight line =

Difference in the value of the slope = %



ROTATING ASSEMBLY

1. vertical shaft
2. cross shaft assembly
3. cord
4. safety cap
5. socket head cap screw
6. counterweight
7. thumb screw
8. spring attach screw
9. knurled nut
10. thumb screw
11. spring
12. test mass
13. eyebolt
14. knurled nut
15. steel washer
16. socket head cap screw

BASE ASSEMBLY

17. base
18. rubber foot (4)
19. slotted head screw (4)
20. bearing housing
21. upper ball bearing
22. lower ball bearing
23. socket head cap screw (3)

MEASUREMENT ASSEMBLY

24. radius indicator
25. ball bearing pulley
26. socket head cap screw
27. spacer
28. hex nut
29. steel washer (2)
30. hex head cap screw (2)