EA-12 Coupled Harmonic Oscillators

Introduction

Owing to its very low friction, an Air Track provides an ideal vehicle for the study of Simple Harmonic Motion (SHM). A simple oscillator assembles with two springs and a single Glider or one spring with the Glider on a tilted Air Track. The parameters of the oscillator can easily vary to investigate the significant variables. Coupled oscillators can be constructed using additional Gliders and springs. Using ceramic magnets attached to the skirt of the Glider you can damp the oscillator. These magnets produce eddy currents in the Air Track, which generate a velocity-dependent damping force. With the addition of the Daedalon EA-40 Precision Sine Drive, you can perform a number of driven harmonic motion experiments.

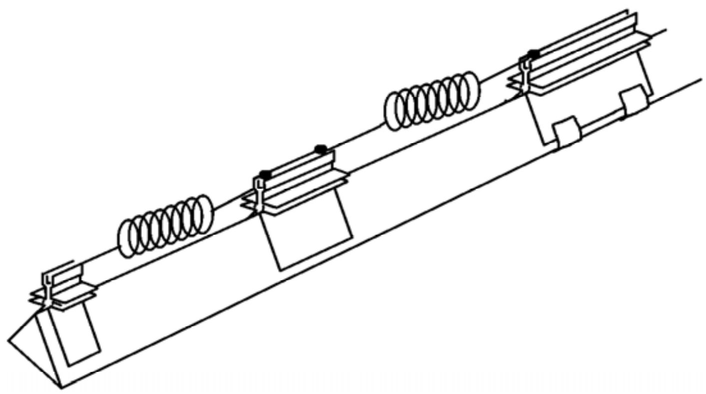

Figure One

Simple Harmonic Motion

Measurement of Period

1. Attach one of the 2.5 cm-long springs to the end stop's top lug.

2. Connect the other end of the spring to the lug on one end of the 150 g Glider. Attach a second spring to the other end of the Glider.

The 2.5 cm springs supplied with the EA-12 Coupled Harmonic Oscillators will stretch to 25 cm (10x) without damage, but that is not long enough to reach the far end stop, so you must construct a temporary end -stop midway along the Air Track.

3. You may assemble the temporary end stop in a number of ways. A simple option is to tape a 450 g Glider to the Air Track and use the lug on the top to hold the end of the spring, as shown in Figure One. Tape the Glider from one skirt under the Track to the skirt on the other side about 25 cm from the Track End Stop. Don't use tape on the top side of the Track to avoid plugging the air holes. Two strips from side to side should be enough to hold it.

4. Turn on the Air Source. The Glider will oscillate back and forth about its equilibrium position. Move it about 5 cm from the center and release it. With a stop-watch, measure the time taken for 20 cycles.

5. Repeat the measurement several times. The frequency is very reproducible.

An ET-40 Electronic Stop Clock and a Photogate measure the period directly, a more rapid assessment. To make this measurement,

6. Attach a small, approximately 1 cm-wide piece of index card in the slot at the top of the Glider. The width is not important since the Clock measures the time that the leading edge of the flag breaks the beam.

7. Place an EA-24 Photogate on the Air Track at the equilibrium point in the Glider's motion. Adjust the height of the Photogate until the flag added to the Glider clears the top of the Gate but breaks the beam.

8. Set the Electronic Stop Clock to its *PERIOD* mode.

9. Displace the Glider about 5 cm as before and release. After a couple of cycles, press the *RESET* button on the Stop Clock. The Clock will measure the time between the first and the third time the Photogate is broken. After the period is displayed on the Clock, stop the Glider.

10. Repeat this measurement several times so that you can estimate the error of the measurement.

The period measured by these two techniques should be very similar to each other. Which measurement has the smallest error? Which measurement do you think is most accurate?

Effect of Mass

One of the determining parameters of the frequency is the mass of the Glider. To investigate this effect:

1. Weigh the Glider without the coil springs and record its weight.

2. Reassemble the springs and Glider and add a small mass to the Glider. The added mass can be a block of metal, such as an EA-15 Riser Block, which will sit on top of the Glider. The sinusoidal motion is gentle enough that the block will sit without falling off.

3. Measure the period as before then weigh the Glider and added mass.

Typical measurements gave these results:

Mass (g)	Frequency (Hz)
140	1.081
164.4	1.012

The frequency of oscillation of a mass and two springs is given by

$$\square\square = 1/2\pi\ (k_o/m)^{1/2}$$

so that

$$\square_1 / \square_2 = (m_2/m_1)^{1/2}$$

From these measurements

$$\square_1 / \square_2 = 1.081 / 1.012 = 1.068$$

and

$$(m_2/m_1)^{1/2} = 1.082$$

which agrees to within 1% and confirms the theory.

Results are more convincing if you measure the period of the Glider with several different masses added to it and plot a curve of the frequency against the square root of the Glider mass, which should produce a straight line.

Effect of Damping

In order to investigate the effect of damping on the harmonic oscillator, tape magnets to the skirt of the Glider. Six ceramic magnets are included in the EA-12 kit. Magnets moving on the Glider induce eddy currents in the conducting surface of the track. The direction of the current resists the movement of the Glider. The magnitude of the current is proportional to the Glider velocity, so the damping force is proportional to the velocity. This is helpful, since the differential equation describing damped oscillations includes a friction term proportional to the velocity. Four magnets can be attached with masking tape, two per side, to the skirt of the Glider.

Keep the magnets apart so their fields don't cancel one another. Remember that the magnets increase the mass of the Glider, so that the resonance frequency will be less than that found in the previous experiment.

The simplest method of measuring the rate of damping of the oscillator is to measure the decrease in amplitude of motion of the Glider with time.

Since

$$x = x_o \exp(-(b/2m)\, t)$$

then

$$\ln x = \ln\ x_o - (b/2m)\, t$$

If the log of the amplitude is plotted against time, the slope of the line gives the damping rate.

To measure the decay of a damped Glider, tape two of the magnets to the skirt of the Glider; the other two remain on top of the web of the Glider as before. Measure the amplitude every five seconds after releasing the Glider. Data for a sample experiment follows. The experiment was repeated with no magnet and four magnets on the skirt, but the data shown are for two-magnet damping

1. The experimental setup is the same as the previous experiment. Tape four magnets to the top of the Glider. They will not produce damping in this position but they will add to the mass. We don't want the mass of the Glider to change as we change the damping.

2. Measure the frequency of the Glider with the magnets on top.

3. Move the Glider 3 cm from its rest position. Measure its position by the rear edge of the Glider on the meter scale on the Track. Release the Glider and start your stop watch.

4. Measure the amplitude of the motion every five seconds for two minutes or until the amplitude is negligible. If two people are cooperating in the experiment, one should call out the time, while the other records the position of the rear edge of the Glider. Don't worry about making small errors in reading the amplitude; they will average out without a serious effect on the measurement of the damping rate.

5. Change the damping magnets on the Glider skirt and repeat the experiment. When taping down two magnets on one side, keep them as far apart as you can so that their fields don't cancel each other.

Typical data are shown in the following table.

For the undamped case, the data were recorded for two minutes. What causes the damping when there are no magnets creating eddy currents? What factors absorb energy from the Glider?

Sample Experiment - 2 Damping Magnets

Time (sec)	Amplitude (cm)	ln(Amplitude)
0	5.0	1.61
5	3.9	1.36
10	2.7	.99
15	2.0	.69
20	1.6	.47
25	1.2	.18
30	.9	-.11
35	.7	-.36
40	.5	-.69
45	.4	-.92
50	.3	-1.20
55	.2	-1.61
60	.2	-1.61

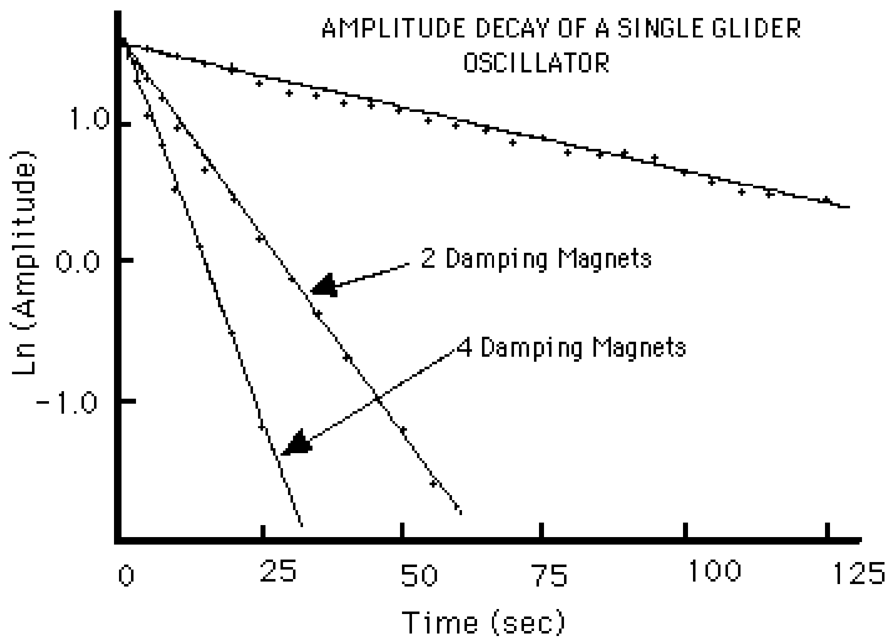

Figure Two

These data are plotted in Figure Two and follow a straight line quite well, verifying that the damping follows the theory and is velocity-dependent. The increased scatter of the points at small amplitudes is due to inaccuracies in measuring the tiny amplitudes.

The time taken for the amplitude to drop by a factor of 2 is the half-life of the oscillator, $T_{1/2}$. Note this value by finding the time required for the ln (amp) to decrease by ln 2 in the curve. In this case $T_{1/2}$ = 72 seconds. Use the half -life of the oscillator to calculate its Q. The Q value is a measure of the quality of the oscillator and is used in electronics as well as with mechanical systems to describe the rate at which the oscillator dissipates energy. Higher values of Q have lower dissipation.

Q can be calculated from:

$$Q = (\square / \ln 2)\,(T_{1/2}/T) = (\square / \ln 2)\,(72/0.94) = 347$$

which is quite high for a mechanical system.

For the damped case, the amplitude drops by a factor of 2 in $T_{1/2}$ = 27 seconds for two magnets, and $T_{1/2}$ = 8.5 seconds for four magnets. This gives values of Q of 130 for two magnets and 41 for four magnets.

Multiple Mass Oscillators

The Air Track equipped with a Sine Drive also performs the measurement of resonances in multiple-mass systems. When two or more Gliders are connected together with springs and driven by the Sine Drive, the resulting system will have a number of resonant peaks equal to the number of Gliders in the system. The motions of the Gliders can be quite complex at resonance, particularly if using several Gliders. For more than two Gliders, the amplitude of motion of each Glider will not be equal for many of the resonances, so that the measured amplitude will depend upon which Glider is used. Finding the resonance frequencies is challenging, particularly for four or more Gliders.

Multiple-mass experiments are analogs for many practical vibration problems and give considerable insight into complex system vibrations. When long strings of Gliders are used, they form a good analog to a crystal model, illustrating the acoustic and optical modes, as well as phenomena such as the ultraviolet cut-off in ionic crystals.

Two Gliders

The simplest multiple Glider system consists of two Gliders and three springs. It has two resonance frequencies, called the symmetric and anti-symmetric modes, which are simply related to each other and to the frequency of a single Glider two-spring oscillator. The frequencies of the symmetric mode is given by:

$$\omega_s = (3\ k_o/m)^{1/2}$$

and the anti-symmetric mode is given by:

$$\omega_a = (k_o/m)^{1/2}$$

For comparison, the frequency of the single Glider oscillator is:

$$\omega = (2\ k_o/m)^{1/2}$$

Check the resonances of the two-Glider oscillator by comparing the first experiment's value with the above calculation.

1. Connect a 2.5 cm spring from the solder lug hole on the top of the left End Stop to the solder lug on the top of one Glider.
2. Connect a second 2.5 cm spring from the other end of the Glider to a second Glider.
3. Connect a third 2.5 cm spring from the other end of this Glider to the solder lug hole on the temporary end stop.
4. Tape one magnet to the skirt of each Glider. If there is no added damping in the system, it takes a long time for the system to settle into a state. Try it with damping at first, then investigate the motion without the magnets.
5. Turn on the air. If the Gliders were centered, they will have little motion when they begin to float.
6. Move both Gliders together 3 cm from their rest position. Release them both at the same time so them will move toward their rest position together. They will move in the symmetric mode of oscillation.
7. After a couple of cycles measure the period. There will be some variability in the period measurement because the motion will not be exactly symmetric.
8. From the Glider rest position, move the Gliders 3 cm in opposite directions, i.e. 6 cm apart from their rest position. Release them and they will move in the antisymmetric mode.

9. Measure the period of the motion.

For two Gliders there are two resonant frequencies. Depending upon the Glider mass and spring constants, the resonances will lie between 0.6 and 1.3 Hz for this experiment. Data from a typical experiment are shown in the following table.

Check the resonant frequencies against the theory. In a typical experiment, where □ is the resonant frequency for a single Glider and since:

$$□_a = □□ / 2^{1/2} \text{ and} □□_s = □□□ \text{ x } (3/2)^{1/2} :$$

	Measured	Calculated
□a	.72 Hz	.72
□□s	1.26 Hz	1.26

Clearly, the check between theory and measurement is very good.

When repeating these experiments, don't be concerned if your resonant frequencies don't match those in the example. The frequencies depend upon k_o, the force constant of the springs, and it is likely that your springs won't be identical to those used in the example.