611-1810 (25-115) Centripetal Force Paradox

Required Equipment (not included):
Motorized turntable (e.g., a record player works well)
Meter stick

Description:
The Centripetal Force Paradox shows a counter-intuitive demonstration where the buoyancy of a bobber causes it to move in the direction of the acceleration. Centripetal is Latin for "center seeking."

Safety:
Always wear safety goggles when performing this experiment. Do not attempt to stop the rotating arm once it is in motion. Allow the arm to stop of its own accord.

Operation:
To use the Centripetal Force Paradox unit, simply unscrew the clear plastic jars and fill them with water. Re-attach the jars to the mounted lid caps. Carefully turn the unit over, and place the hole in the pivot over the shaft that is on the tripod stand. The unit is now ready for operation. Simply hold the base firmly and give it a good spin to see the bobbers in motion.

In order to perform the mathematical calculations outlined on the following page, it is necessary to make the following measurements prior to assembling the apparatus. Record the values on page two of the instructions. Using a meter stick, measure the radius of the arm. Also, measure the height of the plastic jar, radius of the plastic jar and diameter of the plastic ball. Fill the plastic jars with water and reassemble as outlined above. You will calculate the volume and mass of water on page three of the instructions.

Theory:
Inertia is a force that acts on an object in order to keep it moving in a straight line with a constant speed or at rest. Applying a force to an object that is perpendicular to the direction of the object's motion will cause the object to change direction but not speed. Newton's second law of motion predicts that when a net force is applied to an object, its motion will change, or accelerate. Centripetal force causes an object to move in a circle. Additionally, unbalanced forces cause a change in velocity. Velocity is a vector quantity, and it refers to the direction and speed of an object. Force has both a direction and magnitude.

Discussion:
The floating bobber will point in the direction of acceleration. The buoyancy of the bobber acts in the opposite direction of the force of gravity acting down in the medium (i.e., water). In the reference frame of the plastic jar, the inertial force pushes toward the outside. Buoyancy pushes the bobber toward the inside of the plastic jar.

When the plastic jars move in a circular fashion, the water forces the bobber to move toward the inside or center of the jar. This is due to the fact that water has a greater density than the bobber. Because water has a greater density, it has more inertia and it resists the force of the jar pushing it in a circle.

A real life example is that of a rapidly accelerating car. The passenger feels as though he or she is moving backwards into the seat as the seat presses into his or her back. The massive inertia of a passenger's body forces objects with less inertia (i.e., the foam in the seat and air behind the person's back) out of the way as the car moves forward.

Mathematical Calculations:
The force needed to keep an object of mass, m, traveling in a circle of radius, r, at a speed of v, is given by the equation below.

$F_c = m \bullet (v^2/r)$

Centripetal acceleration is given by the following equation:

$a_c = v^2/r$

Therefore, $F_c = m \bullet a_c$.

The period of revolution, T, is the amount of time needed for an object to travel one full circle. Velocity is distance/time. In this instance, distance is the circumference and time is the period. Thus, $v = 2\pi r/T$. We can use this equation to rearrange the force equation above and put it in terms of period rather than velocity.

$F_c = m \bullet (v^2/r)$ and $v = 2\pi r/T$
$F_c = m \bullet (2\pi r/T)^2/r$
$F_c = m \bullet (4\pi^2 r/T^2)$

The quantity $a_c = 4\pi^2 r/T^2$ is the centripetal acceleration.

Since the period is amount of time for the object to travel one full circle or revolution, the reciprocal of the period is the number of revolutions completed per unit time. This is called the frequency. Frequency = 1/T. If the period is measured in minutes, the frequency is measured as the number of revolutions per minute (RPM). The SI unit for time is the second (s), and frequency values are usually expressed as cycles per second. This unit for frequency is given as Hertz (Hz) or s^{-1}.

Procedure:
Place the assembled Centripetal Force Paradox unit with tripod base attached (see operation section), onto the center of a motorized turntable (e.g., record player). Set the turntable to the desired revolutions per minutes. Sample calculations are shown below the table. Record your experimental data in the table.

Circular Motion Paradox:

	Sample Data	**Experimental Data**
Radius of arm (cm)	21	
Height of plastic jar (cm)	6.0	
Radius of plastic jar (cm)	3.0	
Diameter of plastic ball (cm)	2.0	
Radius of arm (m)	0.21	
Height of plastic jar (m)	0.060	
Radius of plastic jar (m)	0.030	
Volume of water (m^3)	0.00016956	
Mass of water (kg)	0.16956	
Velocity (m/s)	1.3188	
Acceleration (m/s^2)	8.282064	
Centripetal force (N)	1.404307	
Total centripetal force (N)	2.808614	

Calculations:
Using the sample data above.

Step 1:
Convert measurements above in cm to measurements in m.
1 m = 100 cm

$$21 \text{ cm x } \frac{1m}{100cm} = 0.21 \text{ m}$$

$$6.0 \text{ cm x } \frac{1m}{100cm} = 0.060 \text{ m}$$

$$3.0 \text{ cm x } \frac{1m}{100cm} = 0.030 \text{ m}$$

Step 2:
Calculate the volume of water in each of the plastic jars. Note: $\pi = 3.14$ in calculations.

Volume (V) = $\pi r^2 h$
r = radius of plastic jar (m)
h = height of plastic jar (m)

$V = \pi(0.030\text{ m})^2(0.060\text{ m})$
$V = 0.00016956\text{ m}^3$

Step 3:
Calculate the mass of water (kg). Note: Density of water at room temperature is approximately 1.00 g/mL.

$0.00016956\text{ m}^3 \text{ x } \frac{100^3 cm^3}{1^3 m^3} \times \frac{1mL}{1cm^3} = 169.56\text{ mL}$

Density = mass/volume
$D = \frac{M}{V}$
D x V = M
1.00 g/mL x 169.56 mL = M
169.56 g = M

$169.56\text{ g x } \frac{1kg}{1{,}000g} = 0.16956\text{ kg}$

Step 4:
Calculate the velocity. Angular velocity, ω, is the rate of change of angular displacement. It is a vector quantity that specifies the rotational speed of an object and the axis about which the object is rotating. Angular velocity is typically expressed in radians/second. Assuming that the spin is 1 revolution per second, this translates into 2π radians/second. Ordinary frequency is given by the following equation.

$\nu = \omega/2\pi$

Therefore, 2π radians/second is equivalent to 1 Hertz (Hz). 1 Hz = s^{-1}.

$v = \frac{2\pi r}{T}$

$f = 1/T$
$f = \nu\ (s^{-1})$

$v = 2\pi r\nu$
$v = 2\pi(0.21\text{ m})(1\text{ s}^{-1})$
$v = 1.3188\text{ m/s}$

Step 5:
Calculate the centripetal acceleration.

$a_c = \frac{4\pi^2 r}{T^2}$

$a_c = \frac{4\pi^2(0.21m)}{1s^2} = 8.282064\text{ m/s}^2$

Step 6:
Calculate the centripetal force. Note: 1 kg • m/s^2 = 1 N.

$$F_c = m\frac{4\pi^2 r}{T^2}$$

$$F_c = 0.16956 \text{ kg } \times \frac{4\pi^2(0.21m)}{1s^2} = 1.404307 \text{ N}$$

Step 7:
Calculate the total centripetal force (N). Because there are two plastic jars with roughly the same mass, the centripetal force is doubled.

1.404307 N x 2 = 2.808614 N

Related Products:

611-1280 Liquid Accelerometer
It's easy to visualize the effects of inertial movement with this low cost device. Horizontal acceleration creates characteristic wave patterns to demonstrate inertial movement. Because liquid moves in the direction opposite to the acceleration, you can actually "see" these waves by watching the pattern created by colored liquid. You can even measure the pattern with the imprinted horizontal and vertical grid.

611-1820 Greek Waiter's Tray
Place a clear container of colored water on the platform of the tray and start swinging from side to side. With practice, you should be able to swing in wide circles and even upside down. The container stays put because net force acting on it is always directed radially, or toward the center. This pins the container to the tray along with its contents.

611-1825 Rotating Candle Apparatus
Another way to look at centripetal force using a candle's flame to show the concept.

Warranty and Parts:

We replace all defective or missing parts free of charge. Additional replacement parts may be ordered. We accept MasterCard, Visa, checks and School P.O.s. All products warranted to be free from defect for 90 days. Does not apply to accident, misuse or normal wear and tear. Intended for children 13 years of age and up. This item is not a toy. It may contain small parts that can be choking hazards. Adult supervision is required.