18700 BICYCLE WHEEL GYROSCOPE

STUDENT NAME:_______________________________________

REQUIRED ACCESSORIES

- (1) Turntable (#18750) or Lab Stool (#18725)
- (1) Cord
- (1) Stopwatch
- Assorted Weights
- Pan Balance or Spring Scale
- Tape and/or Post-It® Note paper

PURPOSE

This wheel serves as part of a gyroscopic apparatus that allows students to predict and observe properties of rotational motion.

SAFETY

While the bicycle wheel gyroscope on its own is not inherently dangerous, it can become dangerous when spinning. While the wheel is spinning, please observe the following safety precautions:

- *An instructor should be present at all times while using this apparatus.*
- When holding the wheel, use both hands and grip handles firmly. When performing the demonstration in Procedure A, use a strong rope that will allow a good grip.
- To stop the wheel from spinning, set the wheel gently on the floor until the friction between the wheel and the floor brings the wheel to a stop.
- *Keep hands out of the spokes of the wheel at all times. Placing hands in the spokes of the wheel while it spins can cause serious injury*.
- Do not drop the wheel while it spins. *If you cannot handle holding the wheel while it spins, ask your instructor to take the wheel from you*.
- *If you are not holding the wheel, stand clear of the wheel. Also stand clear of any person spinning on the turntable with the wheel.*
- Do not use this wheel for anything other than its purpose in this instruction manual.
- Wear appropriate eye protection when using this apparatus.

ASSEMBLY

Slip a finger guard over the handle shaft so that it rests against the grip already in place. Slip the shaft into the wheel bearing. Place the last finger guard over the shaft. Finally, use soap and water to lubricate the remaining grip and the shaft end. Then push the last grip on tightly to hold everything in place. Install the second split ring.

INTRODUCTION

The gyroscope, in its simplest form, is a wheel which spins around an axle. J. B. Léon Foucault invented the gyroscope in 1852, in order to conduct an investigation about the rotation of the Earth. The Earth itself is a spinning gyroscope, and effects of its rotation can be seen in oscillating systems such as the Foucault Pendulum. Rotation of air masses and weather systems are also effects of Earth's spinning motion.

The natural properties exhibited by a spinning gyroscope make it an essential part of many machines. Gyrocompasses, which are used on ships at sea, use an electrically or manually driven gyroscope to provide a consistent reading of true North. (A magnetic compass aboard a ship can be affected by the metal hull of the ship, as well as local fluctuations in the Earth's magnetic field.) Gyroscopes serve as a means of creating or measuring stability in bicycles, spaceships, airplanes, and numerous other vehicles. Gyroscopes are also used in Inertial Guidance Systems for missiles, spacecraft, nuclear submarines and many other scientific and military devices.

CONCEPTS

Gyroscopic motion involves the rotation of a wheel about an axle; commonly, the axis of rotation is an axis through the center of the wheel, perpendicular to the plane of the wheel. The concepts which are central to the gyroscope are listed in detail below.

ANALOGOUS QUANTITIES IN LINEAR AND ROTATIONAL MOTION

Linear and rotational motion share some similar quantities. Below is a chart which details the quantities used in linear and rotational motion, showing the similarities in the formulas and measurements. Quantities in bold can be described as vectors as well as scalars. Although angular position can also be measured with a variety of units such as degrees or revolutions, the preferred unit for angular measurement is radians. Symbols and formulas, as well as units, are in each set of parentheses. Consistency with units is very important: in 1999, NASA lost the $125 million Mars Climate Orbiter because one engineering team used metric units while another used English units. This error was not discovered in time to save the spacecraft from destruction

Quantity	*Linear Motion*	*Angular Motion*
Position	Distance (x; meters)	Angle (θ; radians)
Inertia	Mass (m; kilograms)	Moment of Inertia (or Rotational Inertia) ($I = \Sigma mr^2$ *in general for simplified bodies*; kg m^2; r = radius from axis of rotation)
Velocity	Linear Velocity ($v = \Delta x/\Delta t$; m/s)	Angular Velocity ($\omega = \Delta\theta/\Delta t$; rad/s)
Acceleration	Linear Acceleration ($a = \Delta v/\Delta t$; m/s^2)	Angular Acceleration ($\alpha = \Delta\omega/\Delta t$; rad/s^2)
Force	Linear Force ($\Sigma F = ma$; Newton, kg m/s^2)	Torque ($\tau = I\alpha$; N m)
Momentum	Momentum ($p = mv$; kg m/s)	Angular Momentum ($L = I\omega$; kg rad/s)

As in linear motion, position is always measured in relation to an established reference frame. The frame of reference is arbitrary, and may be selected for the situation being studied. The term 'inertia' signifies the presence of a body with mass on which a force can act: a linear force causes acceleration that is inversely proportional to mass. Similarly, a torque causes angular acceleration that is inversely proportional to moment of inertia. While calculating moment of inertia for objects of various shapes is a complicated mathematical process, formulas for the moment of inertia of some simple objects are defined in most physics textbooks.

MOMENT OF INERTIA

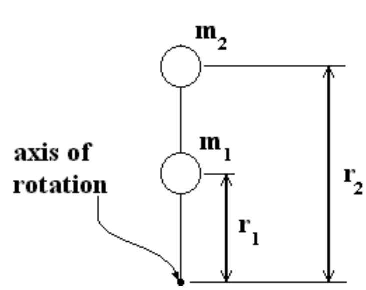
A moment of inertia describes the tendency of objects to resist angular acceleration (to not want to speed up while going around in a circle). Moment of inertia, in this way, is analogous to inertial mass in linear kinematics, where inertial mass is the tendency of objects to resist a linear acceleration (to not want to speed up or change direction while moving in a straight line).

Mathematically, the moment of inertia of an object can be described as

$$(1)\ I = \Sigma mr^2 = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots m_nr_n^2$$

where 'm' is the mass of that object at a particular point within the object, and 'r' is the distance from that point within the object to an arbitrary axis of rotation, and 'n' is the number of points in the object. Moment of inertia has SI units of kg m^2.

Moment of inertia must be calculated for each individual point that makes up the object. Here is an example of a moment of inertia calculation:



The axis of rotation is perpendicular to the page and goes through the lowest point mass. The distance between the center of two adjacent point masses is one meter, and each point mass has a mass of one kilogram for the object above (r_1 = 1 m, m_1 = 1 kg, r_2 = 2 m, m_2 = 1 kg). To calculate the moment of inertia, we use equation (1):

$$I = \Sigma mr^2 = m_1r_1^2 + m_2r_2^2$$
$$= (1\ \text{kg} \cdot (1\ \text{m})^2) + (1\ \text{kg} \cdot (2\ \text{m})^2)$$
$$= 5\ \text{kg m}^2.$$

The moment of inertia for any object is always a sum of the individual moments of inertia of that object. Objects may have simple or complicated moments of inertia, but it is still always a sum. For more complicated objects than the one in this example, ask your instructor or consult your text for a table of common yet more complicated moments of inertia.

The basic formula for moment of inertia, $I = mr^2$, can be used directly for a spinning ring (similar to the bicycle wheel), although having spokes for support and any thickness to the ring will introduce some error. A uniform disc, rotating about the axis through the center of the disc, perpendicular to the plane of the disc, has moment of inertia $I = ½mr^2$.

NEWTON'S LAWS OF ROTATIONAL MOTION

Newton's three Laws of Motion can be applied in rotational reference frames in addition to their application in linear reference frames. Below are Newton's Laws of Motion when applied to a rotating body.

- Newton's 1st Law (Rotational Inertia): Unless acted upon by a net torque, an object will stay at rest or rotate at a constant angular velocity.
- Newton's 2nd Law (Torque): The angular acceleration of a rotating object is directly proportional to the torque exerted on that object, and inversely proportional to the moment of inertia of that object.
- Newton's 3rd Law (Reciprocity): If one object exerts a net torque on a second object, the second object will exert a torque that is equal in magnitude and opposite in direction on the first object.

ANGULAR MOMENTUM AND TORQUE

If a wheel spins with an angular velocity 'ω' (Greek letter 'omega'), and has a moment of inertia 'I', then the angular momentum 'L' of the wheel is

$$L = I\omega.$$

If the angular velocity of the wheel changes with respect to time, then a net torque acts on the wheel. If the wheel has an angular acceleration 'α' (Greek letter 'alpha'), such that

$$\alpha = \Delta\omega/\Delta t,$$

then the torque 'τ' (Greek letter 'tau') exerted on the wheel to cause that angular acceleration is

$$\tau = I\alpha.$$

CONSERVATION OF ANGULAR MOMENTUM

A net torque on a rotating system results in a change in angular momentum, described by the mathematical equation

$$\tau = \Delta L/\Delta t$$

where 'τ' is the net torque on a system, 'ΔL' is the change in the angular momentum of that system, 'Δt' is the time interval in which the torque is applied.

If the net torque on a rotating body is zero, then the change in angular momentum with respect to time is also zero. Therefore, the angular momentum at any point in time remains the same, and angular momentum is said to be conserved. This property of angular momentum is similar to linear momentum, which is conserved if no net force acts on the system.

GYROSCOPIC PRECESSION

Suppose we have a bicycle wheel which spins on an axis through its center, and perpendicular to the plane in which it lies. Through its center, the wheel is mounted to a rod parallel to the axis of rotation.

Now, imagine that this bicycle wheel is supported at one end of the rod. Two forces now act on the wheel: a weight force, which the Earth exerts on the wheel at its center of mass, and a normal force, which the supporting object exerts upward on the wheel at the end of the rod. Theoretically, these forces will cancel one another, which creates no *linear* movement. However, these forces do not act on the same point: one force acts outside of the center of mass, which causes a torque.

Because the bicycle wheel and the support form a closed system, no net torque acts on the system. If no net torque acts on the system, angular momentum must be conserved, and therefore another torque will act in the opposite direction on the wheel in order to 'cancel' the torque on the wheel due to gravity; this torque will cause the wheel to *precess*, or rotate about an external axis, in the direction opposite the spin of the wheel (i.e. if the wheel is spun clockwise, the wheel will precess counterclockwise). The rate of precession is given by the equation

$$\Omega = (m\ g\ l)/L$$

where 'm' is the mass of the wheel, 'g' is acceleration due to gravity, 'l' is the length of one of the handles on the wheel (the moment arm of the torque due to gravity), 'L' is the angular momentum of the spinning wheel, and 'Ω' is the rate of precession of the wheel.

PROCEDURE A (GYROSCOPIC PRECESSION, PART I)

Allow an instructor to give this demonstration.

First, be sure that the bicycle wheel is properly assembled. Attach the support cord or tie a thin rope to the split ring on one of the handles on the bicycle wheel.

Q1. Draw a front-view free-body diagram of the wheel. Is there a net torque on this system when the wheel is not spinning? Explain.

After making sure that the area around the wheel is clear, hold both handles and give the wheel a spin. After spinning the wheel, keep it suspended by holding on to the attached cord. (This experiment works best when the wheel spins as quickly as possible.)

Q2. Before spinning the wheel, predict its motion.

Q3. Was your prediction correct? How would you account for the motion of the wheel after it has begun to spin?

Q4. Predict what would happen if the wheel spins in the opposite direction. Test your prediction, and explain what you observe.

Using a stopwatch, record the amount of time the spinning wheel needs to make one full precession. (Timing two or three full precessions will give the best results.) Repeat this procedure and record several different rates of precession for the wheel.

In order to observe the change in the rate of precession of the wheel as the torque due to the force of gravity changes, add masses in 250 gram increments to the handle that is not attached to the cord. Predict how the rates of precession will change, and time the rate of precession of the wheel as you add mass to the wheel.

Q5. Was your prediction correct?

Q6. How does adding mass to the handle of the wheel affect the rate of precession for the wheel? Explain.

Q7. Observe and record what happens to the rate of precession as the wheel eventually stops spinning.

PROCEDURE B (GYROSCOPIC PRECESSION, PART II)

Allow an instructor to give this demonstration.

(1) Support the bicycle wheel gyroscope from one axle with a string or cord, then support the wheel from the other axle by holding it up manually. Predict the motion of the wheel if the manually supported end of the axle were let go.

Q1. Was your prediction about the motion of the wheel correct? How can you explain the motion of the wheel?

(2) Support the wheel again as described above in step (1). This time, mark the top of the wheel using a 'Post-It'. This piece of paper will designate one point on the wheel, for observation. Again, let go of the manually supported end of the axle.

Q2. Describe the subsequent motion of the 'Post-It'. Where does the 'Post-It' on the wheel want to "fall"? (Hint: use an object in the room where you are using the wheel as a reference point; doorways or windows are good examples of reference points.)

(3) Turn the wheel, moving the 'Post-It' a quarter-turn in the clockwise direction (when wheel is viewed from the manually supported end of the axle). Let go of the manually supported end of the axle, as described above in step (1).

Q3. Which way does the 'Post-It' "fall"?

(4) Now, while holding the wheel, manually pull the point on the wheel where the 'Post-It' is located in the same manner as the 'Post-It' moved in steps (1) and (2). (In other words, if the 'Post-It' rotated towards a back window in your classroom in steps (1) and (2), pull the 'Post-It' in this same direction when it is a quarter-turn away from the top of the wheel.)

Q4. What motion of the wheel would result from pulling this point?

(5) Rotate the 'Post-It' another quarter-turn, as you did in step (3). At this point, the 'Post-It' should be located at the bottom of the wheel, 180 degrees from its initial position. Let go of the manually supported end of the axle.

Q5. Which way does the 'Post-It' "fall"?

Q6. How does your description of the motion of the 'Post-It' in Q5 compare to the motion of the 'Post-It' that you described in Q2?

Q7. What does the motion of the 'Post-It' at the top and the bottom of the wheel tell you about the overall motion of the wheel?

(6) Repeat step (3) for another quarter-turn ('Post-It' located 270 degrees from its initial location). Let go of the manually supported end of the axle.

Q8. How does the 'Post-It' "fall" when it is at this location?

(7) Again, while holding the wheel, manually push the point on the wheel where the 'Post-It' is located in the same manner as the 'Post-It' moved in step (5). (In other words, if the 'Post-It' rotated towards the front door in step (5), push the 'Post-It' in this same direction when it is a quarter-turn away from the bottom of the wheel.)

Q9. What motion does this rotation suggest?

For a graphical representation of this procedure, see Figure 1 below. In this diagram, the acronym 'COM' stands for 'Center of Mass'. The 'star' on the wheel signifies the 'Post-It', which marks the point on the wheel that is analyzed in this procedure.

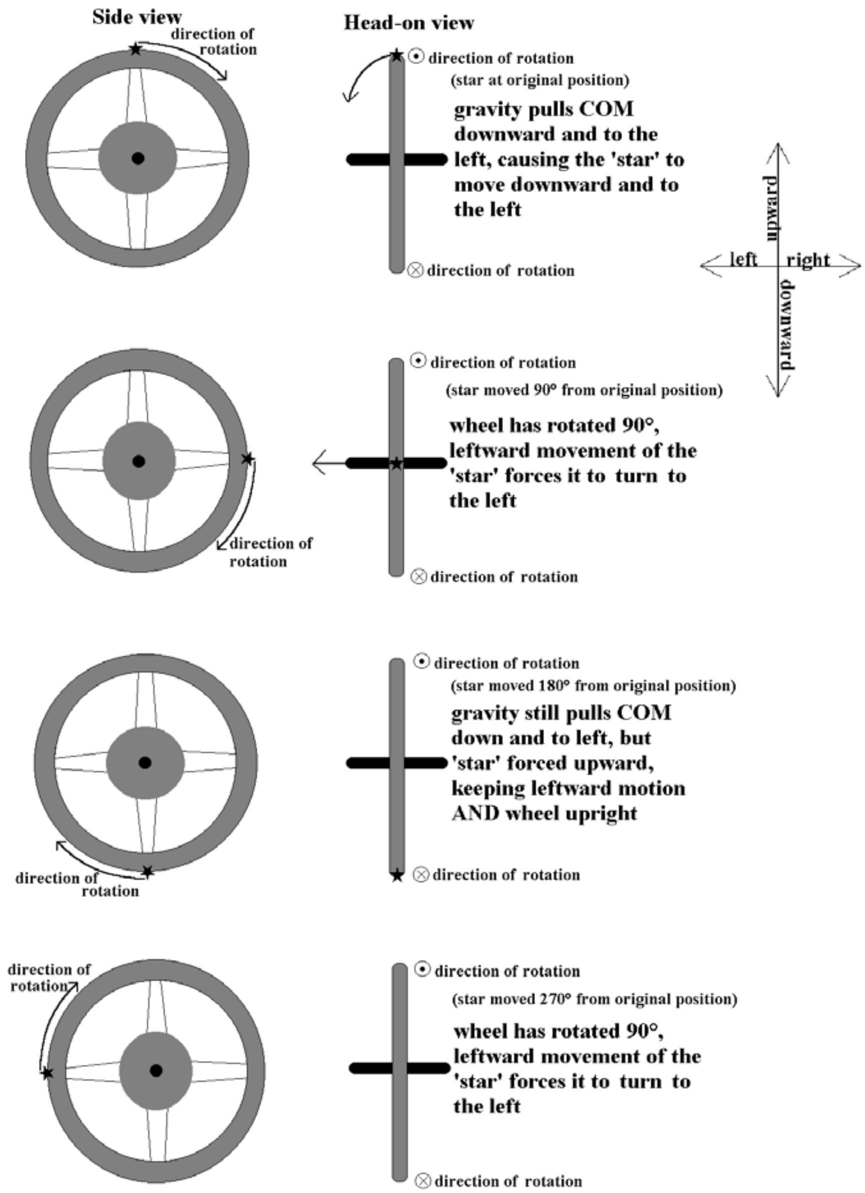


Fig. 1

PROCEDURE C (SPINNING A PERSON)

Place a turntable on a stool and have *one* student sit on the turntable, or place the turntable on the floor and have *one* student stand on the turntable. *Be sure that the student is strong enough to support the weight of the wheel and overcome its rigidity in space. Be sure that the area around the turntable is clear of students and objects!*

After the student sits or stands on the turntable and is properly oriented, have the student hold the bicycle wheel handle at both ends.

Q1. Predict what will happen as the student turns the wheel from side to side.

Give the bicycle wheel a strong spin, and have the student orient the wheel to the left, until the wheel is horizontal.

Q2. Describe the motion of the student on the turntable. Was your prediction correct? How can you account for the student's movement?

Once the student has stopped (with the wheel still spinning and horizontal), have the student rotate the axle 180 degrees in the opposite direction, until the wheel is once again horizontal.

Q3. Describe the motion of the student on the turntable. How can you account for the student's movement?

Q4. Predict how the student on the turntable would rotate if the wheel itself was spinning in the opposite direction.

Repeat this procedure several times, with several different students. In particular, observe the changes in the motion when varying the following:

- Mass of the student on the turntable
- Height of the student on the turntable
- Length of the student's arms
- Speed at which the wheel spins
- Speed at which the student turns the wheel

Q4. How does a change in any of the quantities listed above vary the motion of the student on the turntable?

PROCEDURE D (GYROSCOPIC RESPONSE)

Place a turntable on a stool (or use the Turntable Lab Stool) and have *one* student sit on the turntable, or place the turntable on the floor and have *one* student stand on the turntable. *Be sure that the student is strong enough to support the weight of the wheel and the forces that it may cause. Be sure that the area around the turntable is clear of students and objects!*

After the student sits or stands on the turntable, have the student hold the spinning bicycle wheel, with hands holding each end of the horizontal axle.

Q1. Predict what will happen if you move the student, as opposed to the wheel.

As the wheel is spinning, turn the student on the turntable in the clockwise direction.

Q2. Describe the subsequent behavior of the wheel. Was your prediction correct? What happens as the student comes to a stop?

Q3. Predict what will happen if the student is turned in the opposite direction.

Turn the student on the turntable in the opposite direction.

Q4. Describe the subsequent behavior of the wheel. What do you notice about the motion of the axle of the wheel in comparison to the direction that you turned the student on the turntable?

Return the student to rest on the turntable. Have the student pick a reference point that is in line with the bicycle wheel gyroscope. Repeat the above procedure; for this section, be sure that the top of the bicycle wheel rotates *away from the student*.

Q5. How does the wheel turn in relation to the reference point selected?

Q6. What could be some practical applications of the behavior of gyroscopes demonstrated in this procedure?