18700 BICYCLE WHEEL GYROSCOPE

TEACHER INSTRUCTIONS

REQUIRED ACCESSORIES

- (1) Turntable (#18750) or Lab Stool (#18725)
- (1) Cord
- (1) Stopwatch
- Assorted Weights
- Pan Balance or Spring Scale
- Tape and/or Post-It® Note paper

PURPOSE

This wheel serves as part of a gyroscopic apparatus that allows students to predict and observe properties of rotational motion.

ASSEMBLY

Slip a finger guard over the handle shaft so that it rests against the grip already in place. Slip the shaft into the wheel bearing. Place the last finger guard over the shaft. Finally, use soap and water to lubricate the remaining grip and the shaft end. Then push the last grip on tightly to hold everything in place. Install the second split ring.

TIME REQUIREMENTS

Assembling this product should require no more than 5 to 10 minutes. The demonstrations listed in this instruction sheet should take no more than 20 minutes.
The length of time required to answer discussion questions will vary.

NOTES TO THE INSTRUCTOR:

On The Grade Level of These Activities.

While these demonstrations are suitable for students in grades 5-12, the vector analysis involved in this experiment may only be suitable for college preparatory or Honors level high school physics.

On Required Vector Knowledge.

Students who will use this experiment to study the vector nature of rotational and gyroscopic motion should have a solid understanding of vectors and vector addition. While the experiment itself is good for demonstrations with basic qualitative analysis, questions in this manual require fundamental understanding of the nature of vectors.

On Supervision.

This instruction manual provides excellent demonstrations for a full-class environment, where students can be involved in the demonstrations as a whole. However, the tasks involving control of the wheel, or other rotating apparatuses, should be left to the instructor. The instructor should be present at all times while conducting these demonstrations.

On The Importance of Procedure B.

While a study of torque and angular momentum is important to understanding gyroscopic precession, and accessible for students who understand vectors at that level, Procedure B gives insight about gyroscopic precession to students who have not yet mastered vector analysis. It also provides reinforcement to the vector explanation of gyroscopic precession. This procedure is an important follow-up to Procedure A, and should help to alleviate any confusion that your students will encounter regarding gyroscopic precession.

SUGGESTIONS FOR USE

- Make sure the students that handle the spinning bicycle wheel in Procedures C and D are light enough to experience motion, but strong enough to hold the spinning wheel and overcome its rigidity in space.
- Use the bicycle wheel and turntable on a level surface.
- The best results in any of these demonstrations come when the wheel is spinning as quickly as possible. Experiment with different methods of spinning the wheel, and practice whatever method you find gives the most speed.
- Before conducting any experiments, measure the mass of the wheel using a large pan balance or a spring scale.

STANDARDS

The student will show evidence of the following criteria from the National Science Education Standards (NSES) for grades 5-12:

- Grades 5-12 (**Content Standard A**):
  - Abilities necessary to do scientific inquiry.
  - Understandings about scientific inquiry.
- Grades 5-8 (**Content Standard B**):
  - Motions and Forces. An object that is not being subjected to a force will continue to move at a constant speed and in a straight line.
    (*Addressed in Procedures A and B; this apparatus allows students to observe the types of forces which arise as a result of circular motion.*)
- Grades 9-12 (**Content Standard B**):
  - Motions and Forces. Objects change their motion only when a net force is applied. Laws of motion are used to calculate precisely the effects of forces on the motion of objects. The magnitude of the change in motion can be calculated using the relationship F = ma, which is independent of the nature of the force. Whenever one object exerts force on another, a force equal in magnitude and opposite in direction is exerted on the first object.
    (*Addressed in Procedures A and B; this apparatus allows students to observe the types of forces which arise as a result of circular motion. This apparatus also allows students to explore Newton's Laws of Motion in a rotating reference frame.*)

SAFETY

While the bicycle wheel gyroscope on its own is not inherently dangerous, it can become dangerous when spinning. While the wheel is spinning, please observe the following safety precautions:

- *An instructor should be present at all times while using this apparatus.*
- When holding the wheel, use both hands and grip handles firmly. When performing the demonstration in Procedure A, use a strong rope that will allow a good grip.
- To stop the wheel from spinning, set the wheel gently on the floor until the friction between the wheel and the floor brings the wheel to a stop.
- *Keep hands out of the spokes of the wheel at all times. Placing hands in the spokes of the wheel while it spins can cause serious injury*.
- Do not drop the wheel while it spins. *If you cannot handle holding the wheel while it spins, ask your instructor to take the wheel from you*.
- *If you are not holding the wheel, stand clear of the wheel. Also stand clear of any person spinning on the turntable with the wheel.*
- Do not use this wheel for anything other than its purpose in this instruction manual.
- Wear appropriate eye protection when using this apparatus.

INTRODUCTION

The gyroscope, in its simplest form, is a wheel which spins around an axle. J. B. Léon Foucault invented the gyroscope in 1852, in order to conduct an investigation about the rotation of the Earth. The Earth itself is a spinning gyroscope, and effects of its rotation can be seen in oscillating systems such as the Foucault Pendulum. Rotation of air masses and weather systems are also effects of Earth's spinning motion.

The natural properties exhibited by a spinning gyroscope make it an essential part of many machines. Gyrocompasses, which are used on ships at sea, use an electrically or manually driven gyroscope to provide a consistent reading of true North. (A magnetic compass aboard a ship can be affected by the metal hull of the ship, as well as local fluctuations in the Earth's magnetic field.) Gyroscopes serve as a means of creating or measuring stability in bicycles, spaceships, airplanes, and numerous other vehicles. Gyroscopes are also used in Inertial Guidance Systems for missiles, spacecraft, nuclear submarines and many other scientific and military devices.

CONCEPTS

Gyroscopic motion involves the rotation of a wheel about an axle; commonly, the axis of rotation is an axis through the center of the wheel, perpendicular to the plane of the wheel. The concepts which are central to the gyroscope are listed in detail below.

Analogous Quantities in Linear and Rotational Motion.

Linear and rotational motion share some similar quantities. Below is a chart which details the quantities used in linear and rotational motion, showing the similarities in the formulas and measurements. Quantities in bold can be described as vectors as well as scalars. Although angular position can also be measured with a variety of units such as degrees or revolutions, the preferred unit for angular measurement is radians. Symbols and formulas, as well as units, are in each set of parentheses. Consistency with units is very important: in 1999, NASA lost the $125 million Mars Climate Orbiter because one engineering team used metric units while another used English units. This error was not discovered in time to save the spacecraft from destruction.

Quantity	*Linear Motion*	*Angular Motion*
Position	Distance (x; meters)	Angle (θ; radians)
Inertia	Mass (m; kilograms)	Moment of Inertia (or Rotational Inertia) ($I = \Sigma mr^2$ *in general for simplified bodies*; kg m^2; r = radius from axis of rotation)
Velocity	Linear Velocity ($v = \Delta x/\Delta t$; m/s)	Angular Velocity ($\omega = \Delta\theta/\Delta t$; rad/s)
Acceleration	Linear Acceleration ($a = \Delta v/\Delta t$; m/s^2)	Angular Acceleration ($\alpha = \Delta\omega/\Delta t$; rad/s^2)
Force	Linear Force ($\Sigma F = ma$; Newton, kg m/s^2)	Torque ($\tau = I\alpha$; N m)
Momentum	Momentum ($p = mv$; kg m/s)	Angular Momentum ($L = I\omega$; kg rad/s)

As in linear motion, position is always measured in relation to an established reference frame. The frame of reference is arbitrary, and may be selected for the situation being studied. The term 'inertia' signifies the presence of a body with mass on which a force can act: a linear force causes acceleration that is inversely proportional to mass. Similarly, a torque causes angular acceleration that is inversely proportional to moment of inertia. While calculating moment of inertia for objects of various shapes is a complicated mathematical process, formulas for the moment of inertia of some simple objects are defined in most physics textbooks.

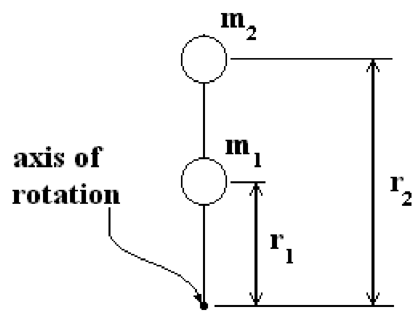
A moment of inertia describes the tendency of objects to resist angular acceleration (to not want to speed up while going around in a circle). Moment of inertia, in this way, is analogous to inertial mass in linear kinematics, where inertial mass is the tendency of objects to resist a linear acceleration (to not want to speed up or change direction while moving in a straight line).

Mathematically, the moment of inertia of an object can be described as

$$(1)\ I = \Sigma mr^2 = m_1r_1^{\ 2} + m_2r_2^{\ 2} + m_3r_3^{\ 2} + \ldots\ m_nr_n^{\ 2}$$

where 'm' is the mass of that object at a particular point within the object, and 'r' is the distance from that point within the object to an arbitrary axis of rotation, and 'n' is the number of points in the object. Moment of inertia has SI units of kg m^2.

Moment of inertia must be calculated for each individual point that makes up the object. Here is an example of a moment of inertia calculation:



The axis of rotation is perpendicular to the page and goes through the lowest point mass. The distance between the center of two adjacent point masses is one meter, and each point mass has a mass of one kilogram for the object above (r_1 = 1 m, m_1 = 1 kg, r_2 = 2 m, m_2 = 1 kg). To calculate the moment of inertia, we use equation (1):

$$I = \Sigma mr^2 = m_1r_1^2 + m_2r_2^2$$
$$= (1\text{ kg} \bullet (1\text{ m})^2) + (1\text{ kg} \bullet (2\text{ m})^2)$$
$$= 5\text{ kg m}^2.$$

The moment of inertia for any object is always a sum of the individual moments of inertia of that object. Objects may have simple or complicated moments of inertia, but it is still always a sum. For more complicated objects than the one in this example, ask your instructor or consult your text for a table of common yet more complicated moments of inertia.

The basic formula for moment of inertia, $I = mr^2$, can be used directly for a spinning ring (similar to the bicycle wheel), although having spokes for support and any thickness to the ring will introduce some error. A uniform disc, rotating about the axis through the center of the disc, perpendicular to the plane of the disc, has moment of inertia $I = ½mr^2$.

NEWTON'S LAWS OF ROTATIONAL MOTION

Newton's three Laws of Motion can be applied in rotational reference frames in addition to their application in linear reference frames. Below are Newton's Laws of Motion when applied to a rotating body.

- Newton's 1st Law (Rotational Inertia): Unless acted upon by a net torque, an object will stay at rest or rotate at a constant angular velocity.
- Newton's 2nd Law (Torque): The angular acceleration of a rotating object is directly proportional to the torque exerted on that object, and inversely proportional to the moment of inertia of that object.
- Newton's 3rd Law (Reciprocity): If one object exerts a net torque on a second object, the second object will exert a torque that is equal in magnitude and opposite in direction on the first object.

ANGULAR MOMENTUM AND TORQUE

If a wheel spins with an angular velocity 'ω' (Greek letter 'omega'), and has a moment of inertia 'I', then the angular momentum 'L' of the wheel is

$$L = I\omega.$$

If the angular velocity of the wheel changes with respect to time, then a net torque acts on the wheel. If the wheel has an angular acceleration 'α' (Greek letter 'alpha'), such that

$$\alpha = \Delta\omega/\Delta t,$$

then the torque 'τ' (Greek letter 'tau') exerted on the wheel to cause that angular acceleration is

$$\tau = I\alpha.$$

CONSERVATION OF ANGULAR MOMENTUM.

A net torque on a rotating system results in a change in angular momentum, described by the mathematical equation

$$\tau = \Delta L/\Delta t$$

where 'τ' is the net torque on a system, 'ΔL' is the change in the angular momentum of that system, 'Δt' is the time interval in which the torque is applied.

If the net torque on a rotating body is zero, then the change in angular momentum with respect to time is also zero. Therefore, the angular momentum at any point in time remains the same, and angular momentum is said to be conserved. This property of angular momentum is similar to linear momentum, which is conserved if no net force acts on the system.

GYROSCOPIC PRECESSION

Suppose we have a bicycle wheel which spins on an axis through its center, and perpendicular to the plane in which it lies. Through its center, the wheel is mounted to a rod parallel to the axis of rotation.

Now, imagine that this bicycle wheel is supported at one end of the rod. Two forces now act on the wheel: a weight force, which the Earth exerts on the wheel at its center of mass, and a normal force, which the supporting object exerts upward on the wheel at the end of the rod. Theoretically, these forces will cancel one another, which creates no *linear* movement. However, these forces do not act on the same point: one force acts outside of the center of mass, which causes a torque.

Because the bicycle wheel and the support form a closed system, no net torque acts on the system. If no net torque acts on the system, angular momentum must be conserved, and therefore another torque will act in the opposite direction on the wheel in order to 'cancel' the torque on the wheel due to gravity; this torque will cause the wheel to *precess*, or rotate about an external axis, in the direction opposite the spin of the wheel (i.e. if the wheel is spun clockwise, the wheel will precess counterclockwise). The rate of precession is given by the equation

$$\Omega = (m\ g\ l)/L$$

where 'm' is the mass of the wheel, 'g' is acceleration due to gravity, 'l' is the length of one of the handles on the wheel (the moment arm of the torque due to gravity), 'L' is the angular momentum of the spinning wheel, and 'Ω' is the rate of precession of the wheel.

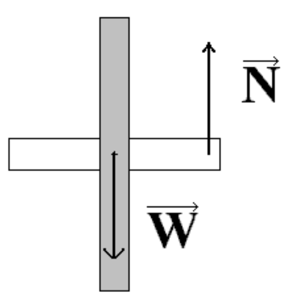
PROCEDURE A (BRIEF DESCRIPTION)

Using the bicycle wheel gyroscope and a cord, either a student or an instructor will demonstrate gyroscopic precession by spinning the bicycle wheel and providing support to one of the handles on the bicycle wheel. Students will measure the rate of precession of the wheel, and will make observations about how that rate of precession changes as the torque due to gravity changes (by hanging weights to the wheel), and as the angular velocity of the wheel changes (by watching the wheel as it stops spinning).

ANSWERS (PROCEDURE A)

Q1. Draw a front-view diagram of the wheel, and draw the forces on the wheel where they act. Is there a net torque on this system when the wheel is not spinning? Explain.

Students should draw a diagram similar to the one below:



where 'W' is the weight force exerted on the wheel by the Earth, and 'N' is the upward normal force exerted on the wheel by the cord. While these forces are equal and opposite, the weight force acts outside of the pivot point at the end of the handle. This will cause a net torque, which will result in a counterclockwise rotation (according to the orientation of the wheel shown in the diagram).

Q2. Before spinning the wheel, predict its motion.

Student predictions will vary.

Q3. Was your prediction correct? How would you account for the motion of the wheel after it has begun to spin?

The wheel should precess opposite the direction of the angular velocity of the wheel. For example, if you spin the wheel clockwise (as seen from the end opposite the cord), the wheel will precess in the counterclockwise direction (as seen from above). This occurs as a result of conservation of angular momentum: because no net torque acts on the system, the torque that comes as a result of the forces acting on the wheel must be countered by a torque in the opposite direction, which causes the precession of the wheel.

Q4. Predict what would happen if the wheel spins in the opposite direction. Test your prediction, and explain what you observe.

If the wheel spins opposite to the direction in the initial demonstration, its angular momentum will also be in the opposite direction. The torque which causes the precession of the wheel still acts in the same direction. Therefore, as a general rule, To conserve angular momentum, the precession of the wheel will also be in the opposite direction compared with the initial demonstration.

Q5. Was your prediction correct?

Students answers will vary.

Q6. How does adding mass to the handle of the wheel affect the rate of precession for the wheel? Explain.

Students should see that the mass on the wheel is directly proportional to the rate of precession of the wheel: as they add more mass to the wheel, it should precess faster.

Q7. Predict what happens to the rate of precession of the wheel as its angular velocity goes to zero. What happens to the rate of precession as the wheel eventually stops spinning? Was your prediction correct?

The angular momentum (and consequently, the angular velocity) of the wheel are inversely proportional to the rate of precession for the wheel, so as the wheel begins to slow down, its rate of precession should continue to increase. Eventually, the wheel will no longer precess as its angular momentum becomes sufficiently low.

PROCEDURE B (BRIEF DESCRIPTION)

Based on their observations of the gyroscopic precession demonstrated in Procedure A, students will qualitatively observe gyroscopic precession by analyzing the wheel at different points during its rotation. This procedure is critical to understanding gyroscopic precession without analysis of torque or conservation of angular momentum.

ANSWERS (PROCEDURE B)

Q1. Was your prediction about the motion of the wheel correct? How can you explain the motion of the wheel?

Student predictions will vary. The wheel should "rotate" towards the ground, about the point where the cord supports the axle of the wheel. The force of gravity acting on the wheel acts at a different point than the force that the cord exerts on the wheel. This causes a net torque, which causes a rotation about the point where the cord supports the wheel (the 'pivot point' for the rotation).

Q2. Describe the subsequent motion of the 'Post-It'. Where does the 'Post-It' on the wheel want to "fall"? (Hint: use an object in the room where you are using the wheel as a reference point; doorways or windows are good examples of reference points.)

The 'Post-It' should "fall" in the same manner as the wheel fell in Q1.

Q3. Which way does the 'Post-It' "fall"?

The 'Post-It' should remain in about the same place as the wheel "falls".

Q4. What motion of the wheel would result from pulling this point?

Pulling the 'Post-It' (now 90 degrees clockwise from the top of the wheel) in the same direction as the wheel fell in step (1) should suggest a precession of the wheel, as demonstrated in Procedure A.

Q5. Which way does the 'Post-It' "fall"?

When the 'Post-It' is at the bottom of the wheel, it should "fall" in the opposite direction as it "fell" in step (1).

Q6. How does your description of the motion of the 'Post-It' in Q5 compare to the motion of the 'Post-It' that you described in Q2?

The motion of the 'Post-It' in Q5 should be opposite the motion of the 'Post-It' in Q2.

Q7. What does the motion of the 'Post-It' at the top and the bottom of the wheel tell you about the overall motion of the wheel?

The motion of the 'Post-It' at the top and the bottom of the wheel should 'cancel' one another, delaying the eventual fall of the wheel.

Q8. How does the 'Post-It' "fall" when it is at this location?

When rotated 270 degrees from the top of the wheel, the 'Post-It' should remain in about the same place as the wheel "falls".

Q9. What motion does this rotation suggest?

When the 'Post-It' is pulled in the same direction as the wheel "fell" in step (5), the wheel should appear to precess in the same direction as it appeared to precess in step (4). These two motions contribute to the overall precession of the wheel.

PROCEDURE C (BRIEF DESCRIPTION)

Either sitting on a stool with a turntable on it, or standing on a turntable which is on the floor, students will use the bicycle wheel gyroscope to experience rotational motion. While on the turntable, students will turn the bicycle wheel gyroscope to either side, and rotate with the turntable as a result of a change in the angular momentum of the wheel. Students will observe how changing the following quantities affects the speed at which the student on the turntable rotates:

- Mass of the student on the turntable
- Height of the student on the turntable
- Length of the student's arms
- Speed at which the wheel spins
- Speed at which the student turns the wheel

ANSWERS (PROCEDURE C)

Q1. Predict what will happen as the student turns the wheel from side to side.

Student predictions will vary.

Q2. Describe the motion of the student on the turntable. Was your prediction correct? How can you account for the student's movement?

Depending on the direction in which the wheel spins, the student on the turntable will either rotate opposite *the direction of his/her turn, or* with *the direction of his/her turn. In either case, the student will experience a rotation if the wheel is turned. This is a consequence of conservation of angular momentum: the angular momentum of the wheel changes as the student turns it, and as a result, his/her angular momentum changes since no net torque acts on the system formed by the student and the wheel.*

Q3. Predict how the student on the turntable would rotate if the wheel was spinning in the opposite direction.

See Q2.

Q4. How does a change in any of the quantities listed above vary the motion of the student on the turntable?

Of the four quantities that can be varied in this experiment, the height of the student is a variable that should make no difference in the motion of the student on the turntable. While the distance from the axis of rotation will make a difference, the height will not make a difference because the student's height measurement is made along the axis of rotation.

Because the mass of the student is related directly to his/her moment of inertia, changing the mass of the student (by students taking turns) will have an effect on the students motion: the higher the mass, the higher the moment of inertia; the higher the moment of inertia, the less angular acceleration due to the change in angular momentum provided by turning the spinning bicycle wheel.

Changing the angular velocity of the wheel, as well as the rate at which the student turns the spinning wheel, will directly change the rate at which the student spins on the turntable. This is because of the direct relation of torque and angular acceleration to a change in angular momentum: as the angular velocity of the spinning wheel increases, so does the angular momentum, and therefore a greater change in angular momentum is required to turn the wheel. As the rate at which the student turns the wheel increases, the change in angular momentum increases, as does the torque exerted on the student by the wheel.

PROCEDURE D (BRIEF DESCRIPTION)

Students will study the response of a gyroscope to external rotation by observing the behavior of the bicycle wheel gyroscope, held by a student on a turntable, as that student is turned. This procedure is designed to demonstrate the importance of gyroscopes in navigational equipment, such as Inertial Guidance Systems or gyrocompasses.

ANSWERS (PROCEDURE D)

Q1. Predict what will happen if you move the student, as opposed to the wheel.

Student predictions will vary.

Q2. Describe the subsequent behavior of the wheel. Was your prediction correct? What happens as the student comes to a stop?

If the student is rotated in a particular direction, the axle of the bicycle wheel will tilt at a right angle to the student's rotation. As the student comes to a stop, the gyroscope will no longer resist the rotation, and it should be 'tilted' in the direction opposite the rotation.

Q3. Predict what will happen if the student is turned in the opposite direction.

Student predictions will vary.

Q4. Describe the subsequent behavior of the wheel. What do you notice about the motion of the axle of the wheel in comparison to the direction that you turned the student on the turntable?

If the student is turned in the opposite direction, the axle of the wheel should change the direction of its 'tilt' accordingly.

Q5. How does the wheel turn in relation to the reference point selected?

The wheel should always tend to tilt back towards the reference point selected by the student as (s)he turns away from that reference point; if the student turns toward the reference point, the wheel will tend to tilt away from the reference point.

Q6. What could be some practical applications of the behavior of gyroscopes demonstrated in this procedure?

This procedure shows the tendency of a gyroscope to maintain its orientation as it spins. This is the fundamental principle behind guidance systems and gyrocompasses.

Revisit Q5, for example: Suppose you have a magnetic compass and you are facing North. If you turn to the East, The compass needle still points North, but North will be to your left, so the needle of the compass will point left. In this procedure, suppose that the reference point you choose is 'true North'. If you turn East (to the right), the gyroscope will tend to turn left, towards 'true North'. This is the primary method of operation for a gyrocompass. Gyrocompasses also exploit the rotation of the Earth to determine true North.