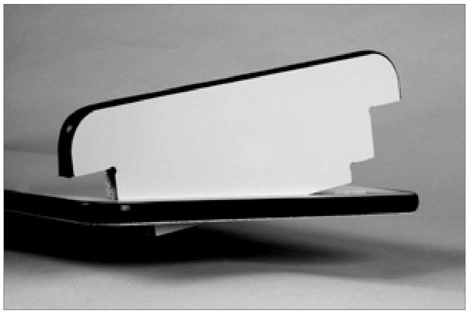
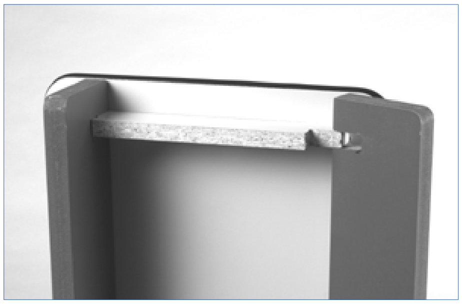
#14660 LEARNING LADDER

Purpose:

To explore the stability of a leaning straight ladder in the context of its footing and load. Also to study the equilibrium of a body acted upon by non-concurrent forces in a plane as illustrated by a ladder resting against a smooth wall.

Required Accessories: Platform balance, Spring Balance, Meter stick, Pliers, Load

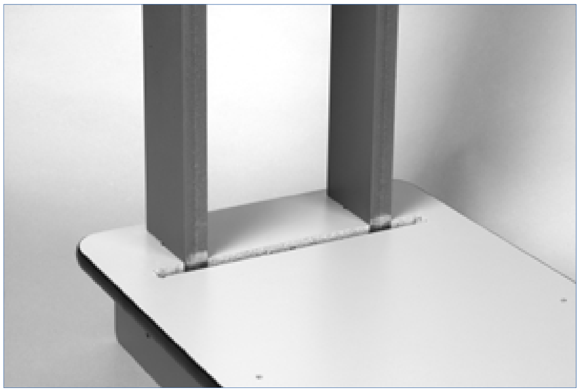


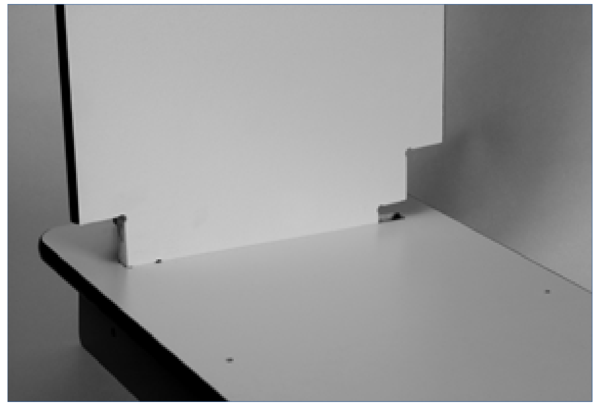


Initial Assembly:

Once all the components have been identified by comparison with the list of contents, the two base beams are set on edge with their notches upward. The base plank is placed on top of the beams, carefully aligning one ot the plank slots with the beam slots using the end that does NOT HAVE the drilled holes. These beams are then held in place by the base end piece, as illustrated above. This completes the fastening of that end of the apparatus. The Base end piece will prevent the ladder or its components from sliding off of the base plank during use.

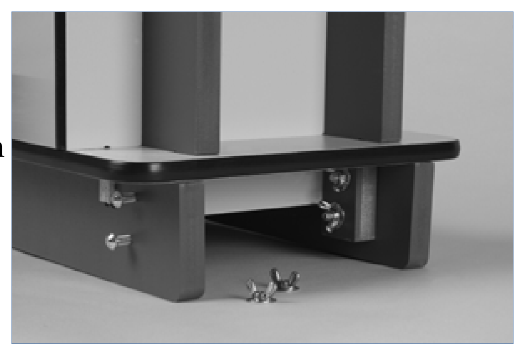
Next, the Vertical (tapered) Beams are inserted through the matching slots in the Base Plank, as shown below. The Vertical Beams will later become attached to the Base Beams using the four bolts and wing nuts. The remaining slot in the Base Plank must remain open to receive the Vertical Plank which serves as the operating wall against which the ladder will lean.



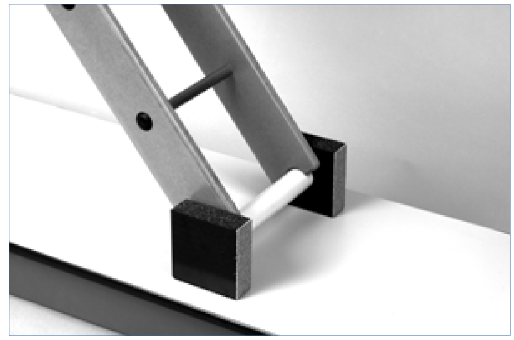


Once the Vertical Plank is fully inserted into the slot of the Base Plank and the slots of the Base Beams, then the Back Plate may be inserted through the slots in the Vertical Beams. Slide the large washer onto the nylon thumbscrew and insert the screw through the hole in the Back Plate, threading it into the Plank.

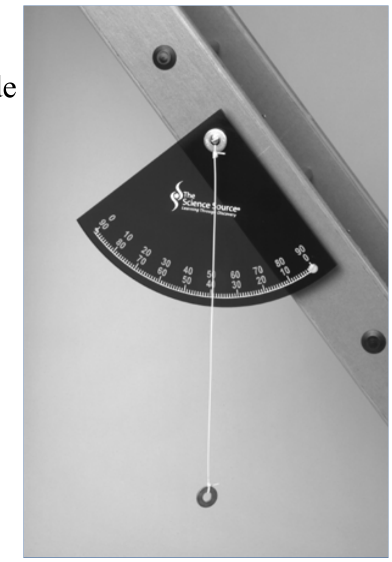
The four bolts and wing nuts can then be tightened in place as shown to the right. These four bolts are the only fasteners used in the whole apparatus, which is now sturdy enough to be used with the ladder and carried from place to place.



The Four-sided Ladder Foot is placed on the Base Plank to support the Ladder while offering four different frictional surfaces to prevent the ladder from sliding.

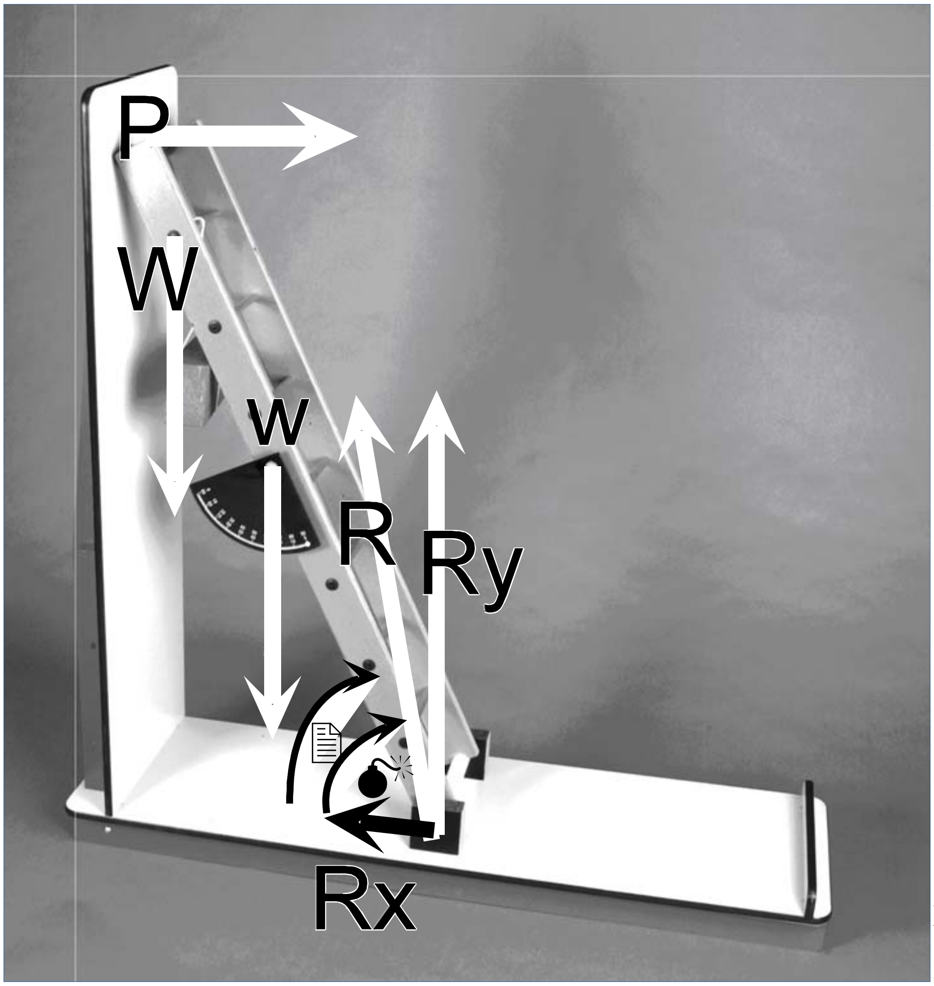


The angle of the Ladder can be read from the mounted Clinometer. This device can easily be fitted to either side of the ladder by moving the lower banana connector to the hole on the opposite end of the degree scale.



A vertical slot is machined in the lower part of the Vertical Plank. This will allow the placement of the eye bolt with two of the wing nuts that are included so as to support a dynamometer spring or spring balance, force probe or other means of measuring the actual thrust of the ladder at various angles of inclination. The view to the left shows how the ladder is inverted for this application, allowing the base of the ladder now to move with trivial friction.

Safety Note:
When a significant load is placed on the ladder when its geometry is such that the four-sided foot will overcome the force of friction, the ladder the foot may slide outward at relatively high speed. While the base end will prevent the foot from leaving the plank, fingers may be pinched between the foot and base end. As a precaution, use the included string to tie the foot to the eye bolt so that its travel is stopped short of the base end.



Background and Operational Details:

An early exploration of the apparatus might be to search out the limiting position of the leaning ladder for each surface provided by the Ladder Foot. Then a certain weight might be suspended from the middle rung and the exploration conducted a second time. Finally, the exploration might be conducted with the same weight attached to each accessible rung to see if that makes a difference. A careful accounting of these findings would put the investigator in the proper frame of mind to carry out some or all of the more detailed procedures that are to follow. For these, the next illustration with its labeled vectors will provide the necessary framework for an essential set of operational equations based on the principles of equilibrium.

Weights can be suspended from any convenient rung using a simple cord as a lanyard. A brick has been shown in some of these views, but conventional slotted or hooked masses would be expected.

Theory of Equilibrium:

For the static equilibrium of a rigid body under the action of forces that are all in one plane, two conditions must be satisfied. First, the vector sum of all of the forces acting on the body must be zero. And secondly, that the sum of all the torques about an axis perpendicular to the plane must also be zero. Each of these conditions can be expressed in the form of simple equations from which the operational principles and procedures can be derived.

For the first condition, since all forces acting in the plane can be expressed in terms of components at right angles to each other, it can be stated that the sum of the vertical components must separately equal zero and the sum of the horizontal components must also equal zero. In a convenient mathematical notation where vertical components are styled as "y" and horizontal components are styled as "x," the conditions can be represented in this way:

$$F_x = 0, \quad F_y = 0 \qquad (1)$$

The second condition is that the sum of the moments of all the forces about any axis perpendicular to the plane of the forces must be zero.

$$\Sigma(F \text{ x } l)_{all} = 0 \qquad (2)$$

In the case of a ladder resting on rough ground and leaning against a smooth vertical wall (as in the previous figure), four forces act upon it, namely: the weight of the ladder **w**, the load **W**, the force at the wall **P**, and the reaction at the ground **R**. Since there is no friction between the ladder and the wall — that is, the wall is smooth — the force at the wall is normal to its surface — in this case horizontal. On the other hand, there is friction between the ladder and the ground — that is, the ground is rough — and the reaction **R** has both vertical and horizontal components, $\mathbf{R_x}$ and $\mathbf{R_y}$.

A table of values would be helpful at this point to begin gathering data that can be resolved by calculation:

Symbol	Meaning	Value
w	weight of ladder	
W	weight of load added to ladder	
P	force at the wall, confirmed by spring balance pulling at ladder top	
R	reaction at the ground	
$\mathbf{R_x}$	horizontal component	
$\mathbf{R_y}$	vertical component	
D 1	distance from center of wheel to balance point (CG) of ladder	
D 2	distance from center of wheel to rung supporting the load	
D 3	distance from center of wheel to end of ladder	
θ	angle the ladder makes with horizontal (from Clinometer/Quadrant)	
φ	angle the resultant R makes with the horizontal	

To simulate the conditions of the ladder problem in a laboratory setting, simply remove the ladder foot and set it aside. Invert the ladder, as pictured on **page 3**, so that it rests on the wheels while being restrained by a lanyard connected to a spring balance or computer force probe, as described and illustrated. When the ladder is in equilibrium, the following conditions apply:

$$F_x = 0, \text{ i.e. } R_x - P = 0 \qquad (3)$$

$$F_y = 0, \text{ i.e. } R_y - w - W = 0 \qquad (4)$$

and taking moments about the center of the wheels,

$$\Sigma(F \times l)_{all} = 0, \text{ i.e. } \mathbf{w * D1 \cos\theta + W * D2 \cos\theta - P * D3 \sin\theta = 0} \qquad (5)$$

The magnitude of R is given by

$$R^2 = R_x^2 + R_y^2 \qquad (6)$$

and its direction by

$$\tan\varphi = R_y / R_x \qquad (7)$$

Time Allocation:

To prepare this product for an experimental trial should take less than thirty minutes. Actual experiments will vary with needs of students and the method of instruction, but individual segments of the investigation are easily concluded within one class period.