14010 Force Table
Student Guide

Recommended Accessories:
- Scissors
- Level (to adjust the legs of the apparatus)
- Blank paper or graph paper
- Pencil
- Ruler
- Protractor
- Calculator

Optional Accessories:
- Paper clips

Purpose:
The purpose of this apparatus it to allow students to test mathematical calculations for equilibrium of forces using weights attached to strings. The strings are, in turn, attached to a center ring, which at equilibrium, is centered around the center post.

Objectives:
To study equilibrium of forces.

Standards: The student will show evidence of the following criteria from the National Science Education Standards (NSES) for grades 5-12:

Grades 5-8 (Content Standard A)- **UNDERSTANDINGS ABOUT SCIENTIFIC INQUIRY**
Mathematics is important in all aspects of scientific inquiry.
Addressed throughout the procedure. Students are asked to use mathematics to predict locations and force necessary for equilibrium.

Grades 5-8 (Content Standard B)- **MOTIONS AND FORCES**
If more than one force acts on an object along a straight line, then the forces will reinforce or cancel one another, depending on their direction and magnitude. Unbalanced forces will cause changes in the speed or direction of an object's motion.
Addressed throughout the procedure. Students are asked to determine force vectors necessary for equilibrium.

Grades 9-12 (Content Standard A)- **ABILITIES NECESSARY TO DO SCIENTIFIC INQUIRY**
USE TECHNOLOGY AND MATHEMATICS TO IMPROVE INVESTIGATIONS AND COMMUNICATIONS
A variety of technologies, such as hand tools, measuring instruments, and calculators, should be an integral component of scientific investigations. ... Mathematics plays an essential role in all aspects of an inquiry. For example, measurement is used for posing questions, formulas are used for developing explanations, and charts and graphs are used for communicating results.
Addressed throughout the procedure. Students are asked to calculate the necessary force vector to achieve equilibrium.

Grades 9-12 (Content Standard A)- **UNDERSTANDINGS ABOUT SCIENTIFIC INQUIRY**
Mathematics is essential in scientific inquiry. Mathematical tools and models guide and improve the posing of

questions, gathering data, constructing explanations and communicating results.
Addressed throughout the procedure. Students are asked to calculate the necessary force vector to achieve equilibrium.

Grades 9-12 (Content Standard B)- **MOTIONS AND FORCES**

Objects change their motion only when a net force is applied. Laws of motion are used to calculate precisely the effects of forces on the motion of objects. The magnitude of the change in motion can be calculated using the relationship F = ma, which is independent of the nature of the force. Whenever one object exerts force on another, a force equal in magnitude and opposite in direction is exerted on the first object.
Addressed throughout the procedure. Students are asked to determine the net force to determine if the forces are in equilibrium. Students are also asked in the assessment questions to use the equation F = ma to calculate the necessary mass to create equilibrium in a situation.

Consult your state learning standards for the appropriate standards addressed using this apparatus in your classroom.

Safety concerns:

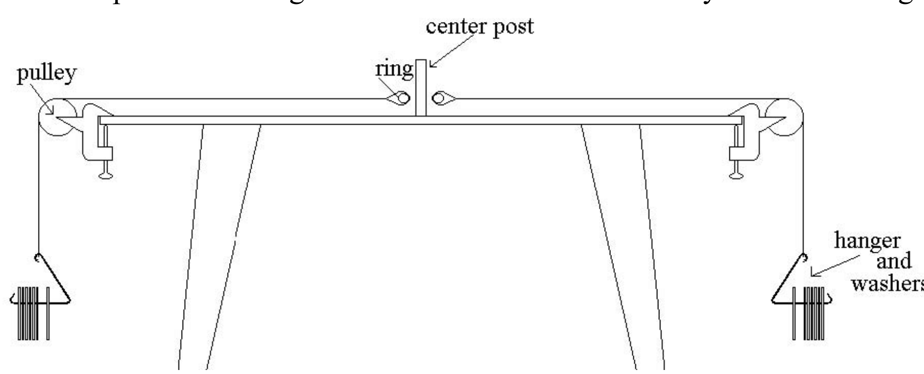
- Eye protection should be worn at all times.
- Washers could pose a slipping hazard if left on the floor. Make sure to keep all washers off of the floor when not in use.
- The table stands approximately 43 cm (17 inches) from floor to top of table. Due to its short size, it can pose a tripping hazard. Make sure to keep tables out of walking paths when not in use.
- The pulleys stick out from the edge of the table. Take care not to run into them, as they may cause injury or break.

Time Requirements:

The assembly of the Force Table takes approximately 5 minutes. The procedure should take between 10 and 30 minutes, due to the need for calculations in predictions of the procedure. Assessment questions may take more time, and may be more suited to homework questions or exam-type questions.

Assembly:

Begin by screwing the three legs into the brackets on the underside of the table top. Push the 3 cm long aluminum rod into the hole in the center of the table until it is flush with the bottom of the table. Check the center ring to make sure it is smooth and free of sharp edges or "snags". (The thread should be able to slide smoothly over the ring without catching.) Cut the thread into 4 equal lengths. Using three of the four lengths of thread, tie one end of each onto the ring. These threads should be tied with loops over the ring to allow the thread to slide freely around the ring. Attach the pulleys to the disk at the desired locations (may need to move them during the experiment). Make sure the thread is short enough so that when the ring is placed on the center post, the hangers do not touch the floor. When all of the threads have been fastened to weight hangers, place the ring over the center post in the table top. Straighten the threads out so they are not tangled or crossed over each other. Lay the thread over the pulleys, as shown in the diagram below. Place the level on the table top and adjust the leveling feet on the legs until the table is level.



Introduction:
Galileo Galilei was born in 1564 and died in 1642. He was an Italian physicist, astronomer, mathematician, and philosopher. Galileo helped invent a better telescope, discovered the first law of motion, and is considered to be the “father” of physics, astronomy, and science, depending upon who you ask. Galileo did much work on the motion of bodies, and much of what is now called Classical Mechanics can be traced back to him. Galileo was the first person to determine the correct mathematical equation for the motion of an object with uniform acceleration (total distance covered is proportional to the square of the time taken), and determined that an object maintains its velocity unless acted upon by a net force, which was contrary to the Aristotelian belief that objects slow down because it was their nature. This later became the important idea behind Newton’s First Law of Motion. Galileo also determined that objects fall towards the earth with the same acceleration, no matter what the mass (also contrary to the Aristotelian belief of the time).

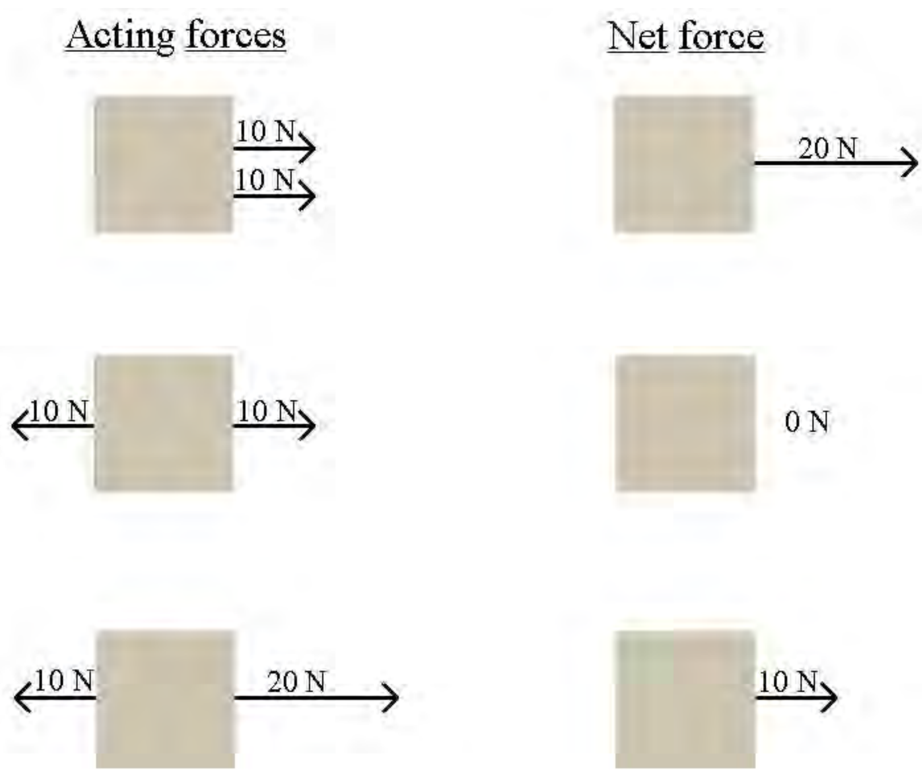
Sir Isaac Newton was born in 1642 and died in 1727. Newton was an English astronomer, physicist, mathematician, philosopher, and alchemist. Newton discovered the three laws of motion that now bear his name. Newton’s First Law states that “An object remains at rest or in uniform straight-line motion unless acted upon by a net force.” Newton’s Second Law states that “the acceleration of an object is proportional to the net force acting on that object and inversely proportional to the mass of the object”. Newton’s Third Law states that “if one object exerts a force on another, the second object exerts a force equal in magnitude but opposite in direction on the first object.” His laws of motion were the basis for Classical Mechanics, and from his laws other laws that govern the universe were derived, such as universal gravitation, the motion of the planets around the sun, and orbit of comets. Newton is also one of two men who independently invented calculus, which has helped scientists do complex mathematics to explain multiple things, like motion of particles, forces, how a ball behaves when it is thrown into the air, and countless other things. Newton also theorized that light was made up of particles, and was the first person to discover that the multiple colors that make up white light are inherent to white light, not added by the prism.

Newton developed methods for predicting the effects of forces in causing acceleration and changes of motion. His Laws of Motion and other discoveries applied to everything from falling boulders to circling planets. Whatever the object, a net force causes changes in motion. Both Galileo and Newton recognized that balanced forces (no net force) result in having an object remain at rest or in uniform motion. This condition of balanced forces is called equilibrium. The force table is a tool that allows you to explore how various forces combine, resulting in equilibrium.

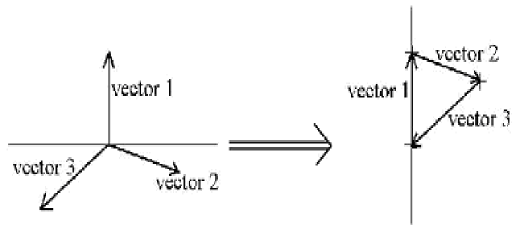
Concepts:
There are many different types of force, such as gravitational, electric, magnetic, mechanical, springs, and friction,to name a few. A force is a push or pull on an object that causes the object to change its motion. The change in motion could be from rest to moving, moving to rest, speeding up, slowing down, or changing direction. Force is a vector, meaning that it has a magnitude and a direction, and should be described using both. Vectors are designated by arrows that have a length representing magnitude of the vector, and point in the direction of the vector. The unit of force is [kilogram]·[meters per second squared] ($kg \cdot m/s^2$), which is known as the newton (N). In general, force can be determined by the equation $\mathbf{F} = m\mathbf{a}$ where ‘**F**’ is the force on the object, ‘m’ is the mass of the object, and ‘**a**’ is the acceleration of the object. (Terms in **bold** are vectors.) This equation is known as Newton’s Second Law of Motion.

When more than one force acts on an object, we think about the net force acting on the object. The net force is determined by adding all force vectors together to get a single vector. Adding vectors can be done a few ways. One way is the graphical method (“head-to-tail” method), and another is the component method. If two equal forces are acting on an object in the same direction, then the net force on the object will be twice that of a single force. If two equal forces are acting on an object in opposite directions, then the net force on the object will be zero. If two unequal forces are acting on an object in opposite directions, then they add together like vectors to make one vector of a magnitude between zero and the larger vector, and in the direction of the larger vector. (For a diagram showing these situations, see below.)

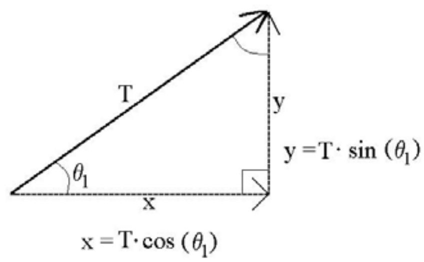
The graphical method (also known as the “head-to-tail” method) of vector addition to discover if forces are in equilibrium is a process in which all vectors are rearranged such that the “tail” (the non-arrow part of the vector) of one vector the meets “head” (the part of the vector with the arrow) of the previous vector. (See diagram below.) Vectors can be moved anywhere necessary as long as their magnitude and direction remain the same.



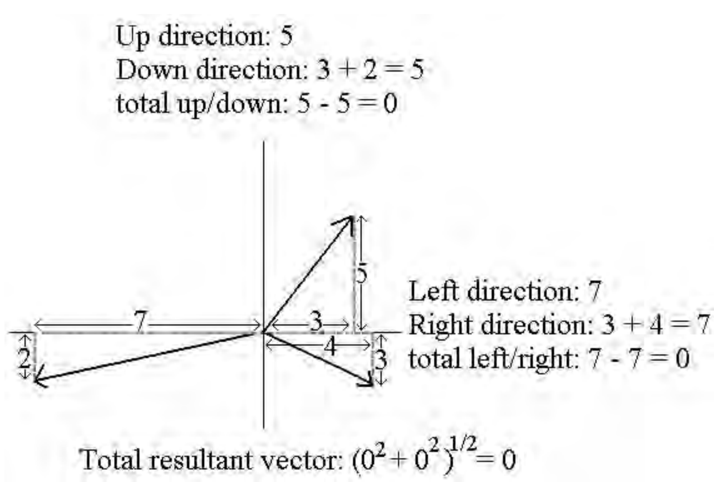
The component method for determining the location and magnitude of an unknown vector to determine equilibrium of forces is a series of steps, detailed below.



1. Determine the horizontal and vertical components of the tension vectors by creating a right triangle and using trigonometric identities, as shown below. (If the vectors are completely horizontal or vertical, then the vector has a component equal to its magnitude in that direction, and a component of zero in the other direction.)

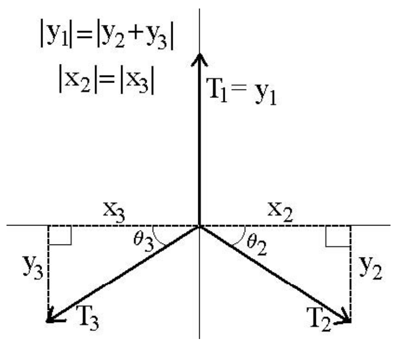


2. Once the components of all known vectors has been determined, the unknown vector will have horizontal and vertical components that, when added to the known components, will yield a resultant of zero. For example, if one vector in the diagram below has a horizontal component of 3 and vertical component of 5, and the other vector has a horizontal component of 7 and vertical component of 2, then the third vector will have a horizontal component of 4 and a vertical component of 3, as shown below.



3. Use the Pythagorean Theorem to determine what the magnitude of the force of the unknown vector will be. The Pythagorean Theorem is: $A^2 + B^2 = C^2$; specifically for this experiment, it can be written as: $\mathbf{T}^2 = x_3^2 + y_3^2$.

4. To determine the angle, use the components of the tension and plug them into the definition of tangent: tan θ = (opposite side/adjacent side). The angle θ_3 will be equal to $\theta_3 = \tan^{-1}(y_3/x_3)$. Subtract this angle from 180° to determine the angle this tension vector must be counterclockwise from the reference angle in order for the forces to be at equilibrium.



5. To determine the force due to gravity, and thus the number of washers necessary, the tension needs to be determined. The tension will have a magnitude as calculated in step 3. For this experiment, the tension force will be equal in magnitude to the weight force. Therefore, $\mathbf{T} = \mathbf{F}_g = m\mathbf{g}$, or $m = \mathbf{T}/\mathbf{g}$, where '**T**' is the tension, '$\mathbf{F}_g$' is the weight force, 'm' is the mass, and '**g**' is the acceleration due to gravity. To determine the number of washers to use, divide m by the mass of a single washer (if mass is being determined by number of washers rather than actual mass of washers, number of washers should be used throughout the calculations as the mass, and then m will be number of washers).

6. If all calculations are done correctly, than the horizontal components of the non-reference forces will cancel, leaving only the vertical components, which should add together to the same magnitude as the reference force vector, but in the opposite direction. The vertical components will all cancel out, leaving an answer of net force equals zero.

Procedure:

Carefully attach the three pulleys to the disk: one at 0°, one at 120°, and one at 240°. The pulleys do not have to be attached extremely tight, only tight enough to keep them from falling off. If they are too tight, they can damage the force table. **Do not put the ring on the post or the thread on the pulleys at this time!**

Q1. Predict what would happen if six washers were placed on each hanger.

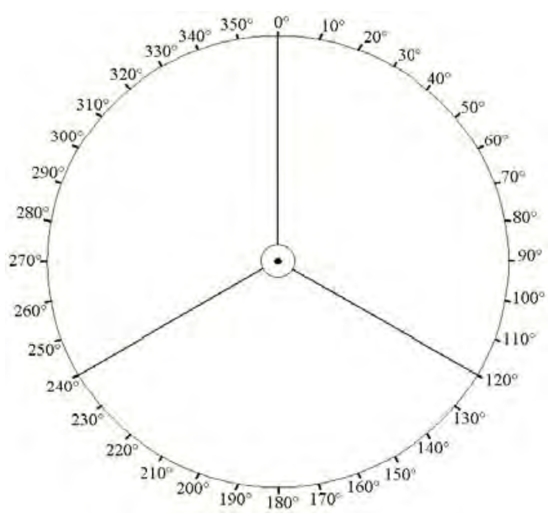
Place six washers on the hangers, and carefully place the ring over the center post. Place the thread over the pulleys, and gently tap the underneath of the table top. (Tapping the table top causes the thread to momentarily not be in contact with the table top, thus allowing the thread to move freely without friction impeding its motion.) Once the ring has stopped moving, observe its location.

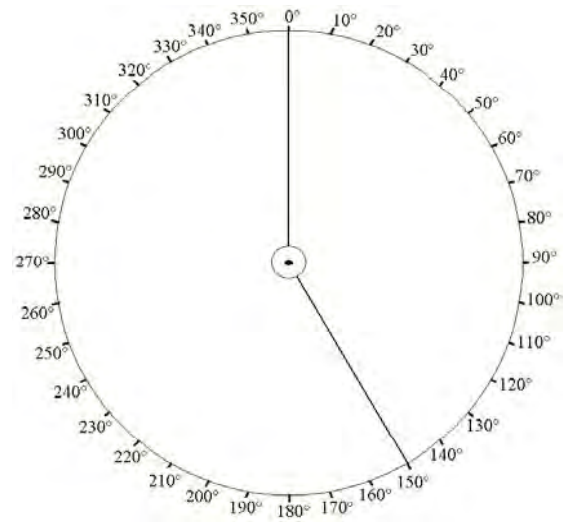
Q2. Where is the center post in relation to the ring?

Q3. Was your prediction correct?

Q4. Using the head-to-tail method, prove the forces are in equilibrium.

Remove the washers from the hangers. Keep one pulley at 0°, and place four washers on the hanger. Move one pulley to 150°, and place six washers on the hanger.





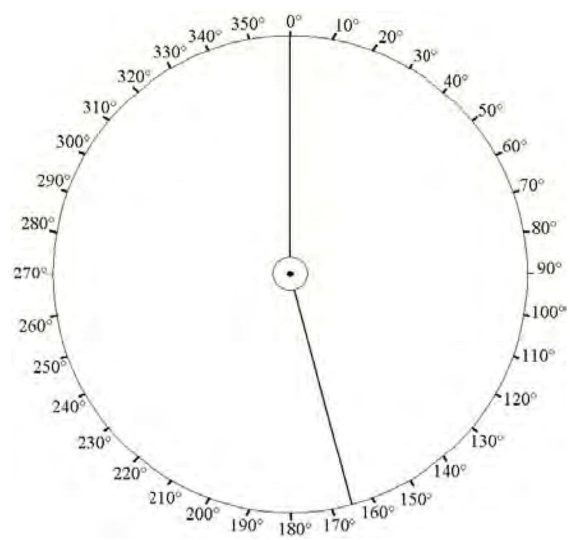
Q5. Predict where the third pulley should be, and how many washers need to be on the hanger for the ring to be centered around the center post.

Move the pulley to the location you have predicted, and place your predicted number of washers on the hanger.

Q6. Was your prediction correct?

Q7. Using the head-to-tail method, prove the forces are in equilibrium.

Remove the washers from the hangers. Keep one pulley at 0°, and place 8 washers on the hanger. Move one pulley



to 165°, and place 3 washers on the hanger.

Q8. Predict where the third pulley should be, and how many washers need to be on the hanger for the ring to be centered around the center post.

Move the pulley to the location you have predicted, and place your predicted number of washers on the hanger.

Q9. Was your prediction correct?

Q10. Using the head-to-tail method, prove the forces are in equilibrium.

Continue the procedure for the following setups (remember to answer all questions):

1. One pulley at 0° with 5 washers, one pulley at 90° with 5 washers.

2. One pulley at 0° with 3 washers, one pulley at 240° with 4 washers.

3. One pulley at 15° with 5 washers, one pulley at 163° with 2 washers.

4. One pulley at 45° with 1 washer, one pulley at 292° with 7 washers.

5. One pulley at 22° with 8 washers, one pulley at 158° with 4 washers.

Assessment:

1. Alice is pushing a box across the floor with a force of 712 N to the right. The force of friction between the box and the floor is 292 N. With what force would Bob have to push to stop the box, and from what direction should Bob push?

2. George is trying to push a washing machine up a ramp into the house. He is pushing with a force of 942 N. Gracie, trying to help George, pulls the washing machine from the top of the ramp with a force of 394 N. The washing machine does not move. What is the force of friction acting on the washing machine? What direction is the friction force pointing in?

3. A student has a force table set up with one pulley at 0° with 10 washers, one pulley at 135° with 8 washers, and has calculated that the last pulley must be put at 210°, and have 14 washers on the hanger. Is the student correct? If so, prove it mathematically. If not, determine where the pulley should be and how many washers are needed.

4. A force table is set up so there is a pulley at 25° and one at 195°, with one free to be placed anywhere. The tension in the thread on the first pulley (25°) is 25 N. The tension in the thread on the second pulley (195°) is 14 N. What is the tension in the final thread and where should the pulley be placed?

5. A force table is set up with one pulley at 42° with a thread of tension 38 N, and another pulley at 252° with a thread of tension 28.5 N. Where must a third pulley be placed for equilibrium to occur, and what mass (in kg) must be placed on a hanger to get the correct tension?

6. Consider a force table set up with four pulleys. One pulley is at 0° with a tension in the thread of 10 N, another pulley is at 90° with a tension in the thread of 10 N, and the third pulley is at 180° with a tension in the thread of 25 N. Where would the fourth pulley have to be placed and what tension would the thread need for the forces to be at equilibrium?

7. A force table is set up with four pulleys- one pulley at 15° with a thread tension of 17 N, one pulley at 160° with a thread tension of 10 N, and one pulley at 350° with a thread tension of 3 N. A student calculates that the fourth

pulley would need to be at 216° with a thread tension of 12 N. Is this student correct? Show all your work.

8. A force table is set up with four pulleys- one pulley at 6° with a thread tension of 18 N, one pulley at 84° with a thread tension of 29 N, and one pulley at 332° with a thread tension of 45 N. What mass (in kg) is necessary to create the appropriate line tension for equilibrium, and where should the pulley be?

Suggestions:

- If desired, paper clips can be used as partial masses (Example- if the mass required is 2.5 washers, the 0.5 washer can be represented with washers to make true equilibrium). A washer weighs approximately 6 g, and a standard paper clip weighs approximately 0.5 g. Students are encouraged to weigh both the washers and the paper clips to determine the exact ratio.
- If desired, hanging masses can be used rather than washers. Every part of the procedure is written with a 20 washer limit, but hanging masses can be substituted if necessary.
- If the pulleys are attached too tightly, then they can damage the table top. Students should be instructed to only tighten the pulleys as much as necessary to keep the pulley from moving, and no more than that. Padding (such as double-sided tape or foam) can be added to the underside of the pulleys to help prevent damage if desired.
- Students should make sure that the thread does not get stuck on the ring, as this can prevent the apparatus from working properly. Make sure students keep the thread out of any divots in the ring.

Brief description of procedure

This procedure allows students to investigate the equilibrium of forces. The procedure begins with students using an equal number of washers at equal angles to each other, and continually increase the difficulty of determining equilibrium, until students have to calculate exact mass and location for multiple pulleys in order to achieve equilibrium.

Because of the knowledge of mathematics necessary for this procedure, it is recommended for students who have taken trigonometry (usually, grades 10-12). Younger grades can use the Force Table; however, modifications to the procedure would be necessary for younger students to be able to use it effectively. In the younger grades, this apparatus may be better served as a demonstration or student exploration with simpler situations, rather than the procedure given.

Other optional demonstrations

An optional addition (which is brought up in the assessment section) is to include a demonstration or additional procedure about four pulleys, as the current procedure only utilizes three pulleys. Students are asked to calculate the necessary vectors for equilibrium in a four-pulley system during assessment questions 4-6.

Answers:

Procedure Answers:

Q1. Predict what would happen if six washers were placed on each hanger.

Answers may vary from student to student.