

## 12500 Air Track -Teacher Manual

### CONTENTS

- *Included Materials*

- (1) Air Track
- (1) Air Source
- (2) 100g Gliders
- (4) Spring Bumpers
- (2) Coil Springs
- (1) Pulley Assembly w/string
- (3) Magnetic Bumpers
- (1) Inelastic Bumper (Banana Plug)
- (4) Connecting Brackets for springs
- (4) 25g Masses
- (2) Thumbscrews

- *Required Accessories*

- Lab Support Stands
- Blocks or Books
- Stopwatch
- Paper Sails
- Photo-gate or Ultrasonic Motion Detector (with data collection software and appropriate changes to apparatus)
- Meter Stick
- Pan Balance or Spring Scale (0-1000 gr.)
- Mass Set
- Protractor

- *Optional Accessories*

- #EA-05 Glider 100g.
- #12010 Glider Launcher

### PURPOSE

To provide a low-friction, one-dimensional surface on which students can predict, observe and record a variety of mechanical concepts.

### TIME REQUIREMENTS

The air track arrives assembled less the attachment of the 3 leveling feet. The individual pieces that supplement the gliders can be attached to the gliders quickly. The full apparatus should require ten to fifteen minutes for complete setup. The experiments listed in this instruction manual should require no more than 30 minutes apiece.

### STANDARDS

The student will show evidence of the following criteria from the National Science Education Standards (NSES) for grades 5-12:

- Grades 5-12 (Content Standard A):
  - **Abilities necessary to do scientific inquiry.**

*(The student uses this apparatus to conduct scientific inquiry about motions, forces, and energy; addressed in Procedures A-H.)*

- **Understandings about scientific inquiry.**

*(The student uses this apparatus to understand the process by which the nature of forces, motion and energy is studied; addressed in Procedures A-H.)*

- Grades 5-8 (Content Standard B):

- **Motions and Forces.** The motion of an object can be described by its position, direction of motion, and speed. That motion can be measured and represented on a graph.

*(Addressed in Procedures A-H; the student uses this apparatus to observe and measure the forces acting on gliders on a near-frictionless environment, and use data collecting equipment to measure motion.)*

- **Motions and Forces.** An object that is not being subjected to a force will continue to move at a constant speed and in a straight line.

*(Addressed in Procedures A-H; the student uses masses and pulleys, springs, or other equipment to exert forces on this apparatus in order to observe Newton's Laws of Motion.)*

- Grades 9-12 (Content Standard B):

- **Motions and Forces.** Objects change their motion only when a net force is applied. Laws of motion are used to calculate precisely the effects of forces on the motion of objects. The magnitude of the change in motion can be calculated using the relationship  $F = ma$ , which is independent of the nature of the force. Whenever one object exerts force on another, a force equal in magnitude and opposite in direction is exerted on the first object.

*(Addressed in Procedures A-H; the student uses masses and pulleys, springs, or other equipment to exert forces on gliders, in order to observe Newton's Laws of Motion.)*

- Grades 9-12 (Content Standard B):

- **Conservation of Energy and the Increase In Disorder.** All energy can be considered to be either kinetic energy, which is the energy of motion; potential energy, which depends on relative position; or energy contained by a field, such as electromagnetic waves.

*(Addressed in Procedure C. Students use this apparatus to perform an experiment involving conservation of mechanical energy.)*

## **SAFETY**

**Please observe the following safety concerns:**

- ***An instructor should be present when this apparatus is in use.***
- To prevent electrical shock, do not use an air track that is broken or has exposed electrical wiring or components.
- Keep hands, arms, and faces off of the air track while it is in use.
- Keep the area around the air track clear.
- If the gliders move too quickly, they may fly off the track. Stand to one side of the air track and be prepared to catch any gliders that may fly off of the track.
- When using pulleys and masses, stand clear of any area where masses may fall. Use a rubber mat to protect the floor and the masses from breaking.
- Wear appropriate eye protection while using this apparatus.

## ASSEMBLY

The air source and track are packaged separately. Carefully remove all parts from the packages and check the contents against the list above.

Insert the track into the triangular air source fitting.



Banana plugs are attached to all bumpers and magnets in this set. To attach these bumpers to the gliders, push the banana clips that are attached to the bumpers into the holes that are located at either end of the gliders.

To attach the included pulley to the air track, insert the banana plug into one of the end stops. Make sure that the pulley wheel spins parallel to the air track.

## SUGGESTIONS FOR USE

- When the air track is not in use, turn it off.
- Leave the air track off when initially setting up a particular experiment.
- Leave the air track off when setting up or resetting the mass/string/pulley setup in Procedure A. The string is easier to manage while air is not flowing through the air track.
- When pushing gliders down the air track, take care not to give the gliders too much speed. Excessive glider speed can damage bumpers or damage the air track. Colliding gliders may also fly off of the air track if they are moving too quickly.

## INTRODUCTION

Every movement in our world is met with some amount of resistance (air resistance, fluid friction, sliding friction, etc.). Forces such as friction and air resistance are known as *non-conservative forces*, because they dissipate kinetic energy of motion. Observations can be made, and measurements can still be taken, in the presence of these ‘limiting’ forces. However, for accurate observations and descriptions of motion in our world, the effects of these non-conservative forces must be accurately included in analyzing the motion. This can add to the complexity of describing and analyzing results.

Experiments conducted in the International Space Station or other spacecraft have the advantage of a low gravity, low friction environment. Most of us are not able to travel to a space station to conduct experiments, and those conditions are not easily recreated on the surface of the Earth. However, the air track can allow motion with extremely low friction, providing much easier observations and results that are more easily understood. For many purposes, when experimenting with the air track, the effects of friction become negligible.

## CONCEPTS

### Equations of Motion.

A cornerstone of classical mechanics are the four *equations of motion*. These equations are simple descriptions of motion in relation to different variables, and are all derived from the equation for displacement as a function of time

$$x = \frac{1}{2}at^2 + v_0t + x_0$$

where ‘x’ is a total displacement, ‘t’ is the time interval in which the motion occurs, ‘a’ is the acceleration of the object, and ‘v<sub>0</sub>’ and ‘x<sub>0</sub>’ are the initial velocity and the initial displacement of the object, respectively. Displacements are measured relative to a certain reference point.

The four equations of motion are:

$$\begin{aligned}(1) \quad v_f &= v_0 + a(\Delta t) \\(2) \quad x &= \frac{1}{2}(v_0 + v_f)(\Delta t) \\(3) \quad x &= \frac{1}{2}a(\Delta t)^2 + v_i(\Delta t) \\(4) \quad v_f^2 &= v_0^2 + 2ax\end{aligned}$$

where ' $v_f$ ' and ' $v_0$ ' are initial and final velocities of an object, ' $a$ ' is the acceleration of that object, ' $x$ ' is the displacement through which the object moves, and ' $\Delta t$ ' is the time interval in

which the object moves. These four equations are the most common ways to mathematically describe motion.

### Newton's Laws of Motion.

*To move an object, one must exert a force on it. Mathematically, forces are described by the equation*

$$\Sigma F = ma$$

where ' $\Sigma F$ ' is the net force on a particular object, ' $m$ ' is the mass of that object, and ' $a$ ' is the acceleration caused by the force. Force and acceleration are both vectors, and both point in the same direction.

Newton's Laws of Motion briefly describe the nature of forces:

- Newton's 1<sup>st</sup> Law (Inertia): Unless acted upon by a net force, an object will remain at rest or at constant velocity.
- Newton's 2<sup>nd</sup> Law ( $\Sigma F = ma$ ): The acceleration of an object is directly proportional to the net force on that object, and inversely proportional to the mass of the object.
- Newton's 3<sup>rd</sup> Law (Reciprocity): When one object exerts a force on a second object, the second object will exert an equal and opposite force on the first object.

### Hooke's Law.

Hooke's Law, which applies to spring-like objects, states that the amount of force required to stretch or compress a spring or other object over a certain displacement is directly proportional to that displacement. Mathematically, this force is described as

$$F = -kx$$

where ' $F$ ' is the force involved in distorting the object, ' $k$ ' is the spring constant of the object (measured in Newtons per meter), and ' $x$ ' is the net displacement of the object, measured from its equilibrium point. A negative sign exists in this equation to show that the direction of the force is always opposite the direction of the displacement of the object.

### Static Equilibrium.

Suppose we have a cart resting on an inclined plane. If we let go of the cart, it will roll down the inclined plane, with either a constant velocity or a constant acceleration.

If the cart rolls down the inclined plane with a constant velocity, or stays in place, then the net force on the cart is zero. This is one example of *static equilibrium*: all of the forces acting on the cart (weight as a result of gravity, friction due to contact between the plane and the cart, and the normal force that the plane exerts on the cart in reaction to gravity) sum to zero. However, if the cart rolls down the ramp with a constant acceleration, static equilibrium is broken: the forces acting on the cart do not sum to zero.

### Work and Energy.

A net force on an object does work on that object. Mathematically, that work is described by the formula

$$W = F d \cos \theta$$

where ‘W’ is the net work done on the object, ‘F’ is the net force on the object, ‘d’ is the displacement through which the object moves, and ‘θ’ (Greek letter ‘theta’) is the angle between the force and the displacement.

Using the equations of motion from linear kinematics, the work-energy theorem can be derived. The work-energy theorem states that any work done on an object will translate to either a change in its kinetic energy, or a change in its potential energy. Mathematically, this theorem is written as

$$W = \Delta KE = -\Delta PE$$

where ‘ΔKE’ is change in kinetic energy, and ‘ΔPE’ is change in potential energy. These energies are defined as

$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ PE &= mgh \end{aligned}$$

where ‘m’ is the mass of a particular object, ‘v’ is the speed/velocity of the object, ‘g’ is the acceleration due to gravity (9.8 m/s<sup>2</sup> locally), and ‘h’ is the height of the object, in relation to a set reference frame.

### Conservation of Linear Momentum.

The linear momentum of an object in motion is given by the formula

$$p = mv$$

where ‘p’ is the linear momentum of the object, ‘m’ is the mass of the object, and ‘v’ is the velocity of the object. Momentum and velocity are both vectors pointing in the same direction.

In all collisions, the sum of the momenta of two objects after they collide is equal to the sum of their momenta before they collide. In an *elastic* collision, the kinetic energy of the objects is the same before and after the collision; kinetic energy is conserved. In an *inelastic* collision, kinetic energy is not conserved; some kinetic energy is lost to friction and heating during the collision. In an explosive event, kinetic energy (from spring potential energy, chemical energy, etc.) increases, although total momentum remains unchanged. For example, when a gun is fired, both the gun and bullet have increased kinetic energy, but their equal and opposite momenta sum to zero.

Conservation of momentum can be mathematically described by the following equation:

$$(m_1v_1)_i + (m_2v_2)_i = (m_1v_1)_f + (m_2v_2)_f$$

where terms on the left side are the initial momenta of two objects in the collision, and terms on the right side are the final momenta of those objects. In an elastic collision, kinetic energy is also conserved:

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

where the terms on the left side of the equation are the initial kinetic energies of the objects in the collision, and the terms on the right side of the equation are the final kinetic energies of the objects in the collision. From these two equations, the velocities of the objects after the collision can be found:

$$\begin{aligned} v_{1f} &= [(m_1 - m_2)/(m_1 + m_2)]v_{1i} + [2m_2/(m_1 + m_2)]v_{2i} \\ v_{2f} &= [2m_1/(m_1 + m_2)]v_{1i} + [(m_2 - m_1)/(m_1 + m_2)]v_{2i} \end{aligned}$$

### Conservation of Mechanical Energy.

Conservation of mechanical energy is an important concept that can be studied with an inclined plane. This concept simply states that on Earth, an object will have a certain potential energy, and when given the opportunity, that potential energy will change to kinetic energy. For example, if you drop a ball off of a building, the ball has a certain potential energy atop that building, as well as a potential energy at the ground. The ball also has a kinetic energy just before it hits the ground, and had no kinetic energy when you dropped it; the change in the kinetic energy of the ball is equal to the change in its potential energy. This translation from potential to kinetic energy, or vice versa, is conservation of mechanical energy.

Mathematically, conservation of energy can be expressed with the following formula:

$$PE_i - PE_f = KE_f - KE_i$$

where  $PE_i$  and  $KE_i$  are the initial potential and kinetic energies of an object, and  $PE_f$  and  $KE_f$  are the final potential and kinetic energies of the object. This formula can be again changed:

$$KE_i + PE_i = KE_f + PE_f.$$

This mathematical manipulation shows that on Earth, under the influence of only gravitational forces, the sum of kinetic and potential energies for an object is the same at all instances of its motion.

### Simple Harmonic Motion.

*Simple harmonic motion*, or SHM, is the periodic oscillatory (back-and-forth) motion of *simple harmonic oscillators*. Systems that undergo simple harmonic motion include pendulums, gliders attached to springs, molecules in a solid, diatomic molecules, and other oscillating systems.

A simple harmonic oscillator experiences a net force which causes its oscillatory motion. For example, the force that causes the oscillation of a pendulum is the gravitational force that the Earth exerts on the pendulum; of course, the pendulum must be displaced from its equilibrium point for oscillation to occur. In the absence of friction and air resistance, the pendulum will oscillate continuously.



For an explanation of the physics behind Procedure G and H, which involves simple harmonic motion, consult the Appendix. The mathematical explanation of these two situations involves knowledge of differential equations, and is intended for a college-level intermediate mechanics class or laboratory.

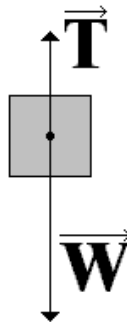
## PROCEDURE A (BRIEF DESCRIPTION)

Using the pulley assembly and hanging masses, students will observe the effects of a constant force on a glider/hanging mass system. Students will use free-body diagrams and their knowledge of forces to calculate the theoretical acceleration of the system in this experiment. Then, students will calculate the acceleration of the system based on data taken from photogates or a sonic motion detector and compare these results to their initial calculations.

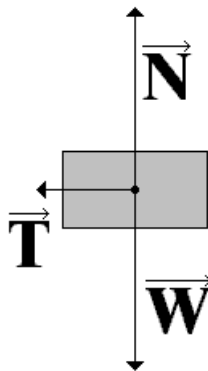
### ANSWERS (PROCEDURE A)

Q1. Draw a free-body diagram for the mass on the pulley and the glider. From these two free-body diagrams, calculate the acceleration of the system.

*Students should draw free-body diagrams that are similar to these:*



*This is the free-body diagram for the hanging mass. First, the string which holds the hanging mass exerts a force on the hanging mass. This force is a result of the tension of the string. The Earth also exerts a weight force on the mass. The net force is equal to the magnitude of the tensile force minus the magnitude of the weight force, and should point downward.*



*Three forces act on the glider: the tensile force from the string (which is equal in magnitude to the tensile force that acts on the hanging weight), the weight force that the Earth exerts, and the normal*



force from the air track. The normal and weight forces are equal in magnitude and opposite in direction, so they add to zero. The only other force acts to the left, so the net force on the glider is in that direction.

The magnitude of the net force on the system is simply the weight force acting on the hanging mass, since the tensile forces are equal and opposite, and the normal and weight forces on the glider sum to zero. However, this force acts on the glider and the mass, not just the mass. So, in order to find the acceleration of the system, calculate the weight force on the hanging mass, and divide that answer by the total mass of the system.

Q2. Use the data you just collected to determine the acceleration of the glider/hanging mass system. How do your results compare to your calculations?

*Student data will vary, but should compare to calculations from the free-body diagram analysis to a reasonable degree of accuracy.*

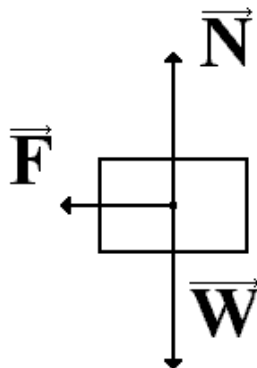
Q3. How can you account for the error in your experiment?

*Friction due to contact between objects and surfaces (string/pulley, glider/surface, etc.), air resistance acting on the glider and the weight, and any possible errors in the measurement of the mass of the glider can result in some error.*

### ANSWERS (ASSESSMENT A)

1. Revisit the free-body diagrams of the individual components of the system that you drew in Q1. Using those free-body diagrams, draw a free-body diagram for the system as a whole.

*The free-body diagram for the system as a whole should look similar to the one below. The tensile forces from the string are equal and opposite, so they will sum to zero.*



2. What forces are in the individual free-body diagrams that aren't in the free-body diagram of the system? Why are they missing?

*The tensile forces from the string are equal and opposite, so they will sum to zero.*

3. How would your results changed if you used a string with a significant mass (i.e. a thick string or cord)?

The acceleration of the system will decrease, since the cord has a significant mass, which is contributed to the overall mass of the system.

4. Calculate the acceleration of the frictionless system below, drawing free-body diagrams as necessary.

While three masses exist in this system, the system will still accelerate uniformly as the mass falls to the floor. Therefore, the free-body diagram drawn in question 1 still applies to this system; the net force on the system is the weight force acting on the mass. For this particular system,

$$\begin{aligned}\Sigma F_{\text{mass}} &= m_{\text{mass}} g \\ &= 9.8 \text{ N.}\end{aligned}$$

To find the net acceleration on the system, replace the mass of the falling object with the weight of the system as a whole. The weight force on the hanging mass is also the net force on the system, so

$$\Sigma F_{\text{system}} = m_{\text{system}} a = 9.8 \text{ N.}$$

The total mass of the system adds to 6 kilograms. The acceleration of the system is therefore  $.61 \text{ m/s}^2$ .

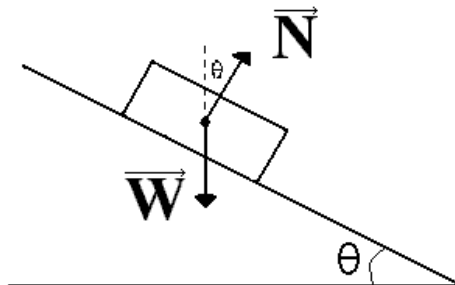
## PROCEDURE B (BRIEF DESCRIPTION)

Students will use the air track as an inclined plane in this experiment, in order to observe a glider while it moves down an incline. First, students will use free-body diagrams and their knowledge of forces to calculate the acceleration of the glider on the inclined air track. Then, students will measure the actual acceleration of the glider using photogates or a sonic motion detector and compare these results to their initial calculations.

## ANSWERS (PROCEDURE B)

Q1. Draw a free-body diagram of the glider at the top of the air track. Leave out friction and air resistance.

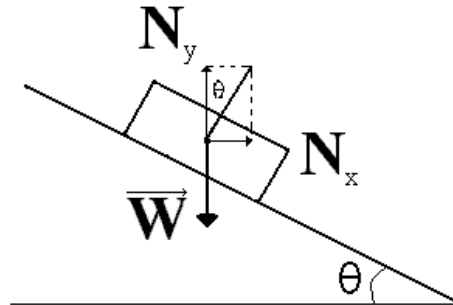
Students should draw a free-body diagram similar to the following:



where 'W' is the weight force exerted on the glider by the Earth, 'N' is the force exerted on the glider by the air track, and 'θ' (Greek letter 'theta') is the angle between the air track and the table. The normal force is equal in magnitude to the weight force. The net force on the glider acts down the inclined air track.

Q2. Using the force vectors from your free-body diagram and trigonometry, calculate the acceleration of the glider on the air track.

*Student answers will vary based on the angle at which their track is inclined. First, break the forces on the free-body diagram into components, as shown below:*



where ' $N_x$ ' and ' $N_y$ ' are the horizontal and vertical components of the normal force.

These components can be calculated using the following formulas:

$$N_x = N \sin \theta$$

$$N_y = N \cos \theta.$$

Notice that the use of trigonometric functions in these formulas depends on the placement of ' $\theta$ '.

Next, add like force components, and once those components are added, divide the resultant force by the mass of the glider to find the acceleration of the glider.

Q3. Why is it important to choose one side of the glider when performing your measurements?

*Choosing one point from which to measure the position of the glider will ensure consistency in your measurements; if you make one measurement from one end of the glider, and the next measure from the other end of the glider, your measurements will be off by one glider length.*

Q4. What assumption do you make about the initial velocity of this system?

*When using the Equations of Motion to calculate the acceleration of the system, assume that the glider has zero velocity at the top of the ramp.*

Q5. Were the results you obtained in this experiment correct? How can you account for any error in this experiment?

*Results may vary. Because measurements from photogates or motion detectors will be relatively accurate if taken correctly, students can assume that air resistance plays a small role in altering their results. Friction is negligible in this experiment due to the nature of the surface.*

### ANSWERS (ASSESSMENT B)

1. If the air track were inclined to an angle of 90 degrees, what acceleration would the glider experience?

*If the air track stood at 90 degrees to the tabletop, the glider would no longer rest on the track, and it would fall at  $9.8 \text{ m/s}^2$ .*

2. If the air track lays flat, how would the glider accelerate?

*If the air track lays completely flat (i.e. at an angle of zero degrees to the tabletop), the glider would experience no acceleration.*

3. Based on questions 1 and 2, the free-body diagrams that you drew during the experiment, and your experimental results, write a mathematical rule that relates the acceleration of the glider to the angle at which the air track is inclined.

*If the acceleration of the glider is  $9.8 \text{ m/s}^2$  at 90 degrees to the tabletop, and there is no acceleration when the air track is flat, the acceleration as a function of angle should involve a function that has its maximum at  $\theta = 90$  degrees. Sine functions have this property, so we can write the following formula relating acceleration to the angle at which the air track is inclined:*

$$a = g \sin \theta$$

*where 'a' is the acceleration due to the inclined air track, 'g' is the acceleration due to gravity, and 'θ' is the angle between the air track and the tabletop.*

## **PROCEDURE C (BRIEF DESCRIPTION)**

Students will continue Procedure C by observing the motion of a glider while it moves up an incline.

### ANSWERS (PROCEDURE C)

Q1. Does the glider move with a constant velocity or a constant acceleration? How can you tell?

*Students should notice that the glider slows down as it moves up the inclined air track; this motion is indicative of a constant acceleration.*

Q2. Draw a free-body diagram of the glider at some point on the air track (neglect friction and air resistance). Would this free-body diagram look any different if the glider is at any other point on the air track?

*Neglecting friction and air resistance, students should draw the same free-body diagram as they would have drawn in Q1 of Procedure B. This free-body diagram will be the same for all points on the air track.*

Q3. Does the data you took confirm your answer to Q1?

*Measurements from the photogates should indicate a constant acceleration. If using a motion detector, create a velocity graph for the glider with respect to time. This plot should appear to be linear on any data collection software, and should have a negative slope.*

Q4. Is your data consistent with the free-body diagram you drew in Q2?

*The data indicates a constant acceleration, which also indicates a constant net force on the glider. This result is consistent with the answer to Q2.*

Q5. Does a change any of these variables affect the results of your experiment? Explain why or why not.

*Because the results for the net force on the glider depend on 'g', the acceleration due to the Earth's gravity (a constant), the mass of the glider should not matter when calculating the acceleration down the inclined air track. However, a change in the angle will affect results: if the air track is elevated to a 90 degree angle to the table, for instance, acceleration will simply be 'g'; if the air track is lowered to rest parallel to the table, the glider will not accelerate.*

### ANSWERS (ASSESSMENT C)

1. As the glider moves up the inclined air track, eventually, it will turn around at the top of the air track and head back down. Does the acceleration stay constant throughout the glider's motion?

*Although the direction of the motion of the glider changes when it reaches the peak of its motion, the acceleration of the glider is constant as it moves.*

2. If the air track were inclined to 90 degrees, what acceleration would the glider experience?

*See Assessment B, question 1.*

3. If the air track is laid parallel to the tabletop, what acceleration would the glider experience?

*See Assessment B, question 2.*

4. Based on questions 2 and 3 and your experimental results, write a mathematical rule that relates the acceleration of the glider to the angle at which the air track is inclined.

*See Assessment B, question 3.*

5. Does the acceleration of the glider down the air track depend on the mass of the glider when friction is not present? Explain why or why not.

*Because the acceleration due to gravity is a constant, and affects all masses identically, the mass of the glider does not affect its acceleration down the air track.*

## **PROCEDURE D (BRIEF DESCRIPTION)**

Students will once again study the inclined air track; this time, they will determine the acceleration of a glider down the inclined air track using energy calculations based on their data. Students will verify their data using conservation of mechanical energy, and will also observe that the acceleration down an incline is mass independent if the surface is frictionless.

ANSWERS (PROCEDURE D)

Q1. Find the potential energy of the glider at the top of the inclined air track and the bottom of the inclined air track. What coordinate system should you use for these measurements?

*In order to make calculations easy, students should use the tabletop as their measurement reference (i.e. let  $h = 0$  m at the tabletop). The potential energy at either point will be the product of the mass of the object, the acceleration due to gravity, and the height of the object relative to the tabletop.*

Q2. Using the Conservation of Mechanical Energy formulas in the Concepts section, calculate the speed of the glider at the bottom of the ramp. What do you assume about the speed of the glider at the top of the ramp?

*Students calculations may vary. The speed of the glider at the bottom of the ramp is*

$$v_f = \sqrt{(2g\Delta h)}$$

*where ' $\Delta h$ ' is the difference between the initial and final heights of the glider. Students should assume that the speed of the glider at the top of the ramp is zero.*

Q3. From your data, find the speed of the glider at the bottom of the ramp. How does your data compare to your initial calculations?

*Student calculations of data will vary, but should be close to theoretical calculations based on conservation of energy.*

Q4. Does the mass of the glider matter in your calculations for the speed at the bottom of the track?

*Because the mass of the glider does not change, and its mass appears in every energy term in the conservation of energy equation, mass will not play a factor in these calculations.*

Q5. From the speed of the glider at the end of the air track and the length of the air track, calculate the acceleration of the glider. Does your result match the results of Procedures B and C?

*This calculation can come from the Equations of Motion in the Concepts section. The calculation should match the results of Procedures B and C if the air track is inclined to the same angle for each procedure.*

ANSWERS (ASSESSMENT D)

1a. What is the speed of object 1 when it gets to the bottom of the ramp?

*To find the speed of object 1, first use the conservation of energy formula. The kinetic energy at the top of the ramp is zero, and the potential energy at the bottom of the ramp is also zero. Therefore,*

$$KE_{\text{final}} = PE_{\text{initial}}$$

$$\frac{1}{2}mv_f^2 = mgh.$$

From that equation, we can find the velocity at the bottom of the ramp:

$$v_f = \sqrt{2gh}$$

Plug in the value of the initial height of the object ( $h = 4 \text{ m}$ ), and get the answer.

$$v_f = \sqrt{2g(4)} = 8.85 \text{ m/s}$$

1b. What is the speed of object 2 just before it hits the ground?

To find the final speed of object 2, the method in question 1 can also be used. Incidentally, the speed of object 2 just before it hits the ground is also 8.85 m/s.

1c. How long does it take object 1 to reach the bottom of the ramp?

Use equation (4) of the Equations of Motion under the Concepts section to solve this problem.

$$v_f^2 = v_0^2 + 2ax$$

Since  $v_0$  is zero in this problem, and 'x' is simply the length of the ramp ( $x = 8 \text{ m}$  in this problem, due to the geometry), the acceleration of the glider can be found:

$$(8.85 \text{ m/s})^2 = 2a(8)$$

$$a = 4.9 \text{ m/s}^2.$$

Notice that the acceleration in this case is half the acceleration due to gravity ( $9.8 \text{ m/s}^2$ ), and that the sine of the angle at which the ramp is inclined is equal to  $\frac{1}{2}$ .

Now, use equation (1) to find the amount of time needed to reach the ground.

$$d = \frac{1}{2}at^2 + v_0t$$

Since  $v_0$  is equal to zero, the equation becomes

$$8 = \frac{1}{2}(4.9 \text{ m/s}^2)(t^2)$$

$$t = 1.81 \text{ s.}$$

1d. How long does it take object 2 to reach the ground?

For this part of the problem, we can simply use equation (1) in the Equations of Motion section:

$$v_f = v_0 + a(\Delta t).$$

In this equation, ' $v_f$ ' is 8.85 m/s, as calculated in 1b; ' $v_0$ ' is zero because the object starts from rest; ' $a$ ' in this situation is just 'g', or  $9.8 \text{ m/s}^2$ . Plug in the values, and get the solution:



$$8.85 \text{ m/s} = (9.8 \text{ m/s}^2)(\Delta t)$$

$$\Delta t = .903 \text{ s}$$

*Notice that the falling object takes half the time to reach the ground as the object on the incline; this is consistent with the fact that the acceleration due to the incline is half of the acceleration due to gravity.*

## **PROCEDURE E (BRIEF DESCRIPTION)**

In this experiment, students will use three gliders (two of identical mass) and either spring bumpers or magnetic bumpers to observe different elastic collisions. Students will use their observations of these elastic collisions to make conclusions about Newton's 3<sup>rd</sup> Law and conservation of linear momentum. Students can also try different collisions than the ones described in the procedure. **(Note: Two gliders are provided with the air track. Additional gliders are needed to complete this and subsequent experiments and may be purchased from Science First, Part #EA-05 Small Glider.)**

## ANSWERS (PROCEDURE E)

Q1. How does each glider move after a 'head-on' collision occurs, where the gliders move toward each other with the same speed?

*Student predictions may vary.*

Q2. Describe the behavior of the two gliders after the collision. Was your prediction correct?

*Due to conservation of momentum, the two gliders should move in opposite directions after the collision occurs, and should move with the same speed. The sum of the momentum before and after the collision should sum to zero.*

Q3. Is the collision you observed consistent with Newton's 3<sup>rd</sup> Law? Explain.

*Due to Newton's 3<sup>rd</sup> Law, the gliders will exert equal and opposite forces on one another. Because the gliders have the same mass, each glider will experience accelerations of the same magnitude, and in the opposite direction. This indeed occurs in the observed collision.*

Q4. Predict the motion of the gliders after this collision occurs.

*Student predictions may vary.*

Q5. Test your prediction, and describe the behavior of the two gliders after the collision. Was your prediction correct?

*The glider in motion should stop when it collides with the glider at rest, and should appear to 'transfer' all of its momentum to the glider at rest.*

Q6. Is the collision you observed consistent with Newton's 3<sup>rd</sup> Law? Explain.

*Both gliders have the same mass; the glider that initially is in motion collides with the stationary glider and stops, while the glider at rest moves in the same direction, with the same speed, as the glider that moves initially. This result is consistent with Newton's 3<sup>rd</sup> Law; the glider in motion experiences an acceleration that is equal in magnitude and opposite in direction to the acceleration that the glider at rest experiences, because the acceleration results in the same change in speed during the collision.*

Q7. Predict the motion of the gliders after this collision occurs.

*Student predictions may vary.*

Q8. Test your prediction, and describe the behavior of the two gliders after the collision. Was your prediction correct?

*While both gliders are moving with the same speed before the collision, the gliders react differently during the collision; the more massive glider should rebound from the collision with less speed than the glider with less mass.*

Q9. Is the collision you observed consistent with Newton's 3<sup>rd</sup> Law? Explain.

*The collision described in Q8 is consistent with Newton's 3<sup>rd</sup> Law: although both gliders exert the same force on the other, the glider with more mass experiences less acceleration than the less massive glider.*

### ANSWERS (ASSESSMENT E)

1a. Find the initial momentum of the system.

*The initial momentum of the system is given by*

$$p_{\text{initial}} = m_1v_{1i} + m_2v_{2i}.$$

*With the values given, and the assigned coordinate system,*

$$p_{\text{initial}} = (875 \text{ kg})(20 \text{ m/s}) - (2150 \text{ kg})(5 \text{ m/s}) = 6750 \text{ kg m/s}.$$

1b. Find the final velocities of both vehicles in this system.

*With the equations given in the Conservation of Linear Momentum portion of the Concepts section, the final velocities of both vehicles can be found:*

$$v_{1f} = [(875 \text{ kg} - 2150 \text{ kg}) / (875 \text{ kg} + 2150 \text{ kg})](20 \text{ m/s}) + [2(2150 \text{ kg}) / (875 \text{ kg} + 2150 \text{ kg})](-5 \text{ m/s})$$

$$v_{1f} = -15.50 \text{ m/s (negative - to the left)}$$

$$v_{2f} = [2(875 \text{ kg})/(875 \text{ kg} + 2150 \text{ kg})(20 \text{ m/s}) + [(2150 \text{ kg} - 875 \text{ kg})/(2150 \text{ kg} + 875 \text{ kg})](-5 \text{ m/s})$$

$$v_{2f} = 9.50 \text{ m/s (positive - to the right)}$$

1c. Check your calculations to make sure that momentum in this system is conserved.

*Now, find the momentum of each vehicle after the collision, based on your calculations for their new velocities. Aside from any rounding, the two total momenta should be equal.*

## PROCEDURE F (BRIEF DESCRIPTION)

This procedure is similar to Procedure E; however, in this procedure, students will create elastic collisions and examine them using quantitative data as well as observations. Particularly, students will observe both conservation of momentum and conservation of kinetic energy in elastic collisions.

### ANSWERS (PROCEDURE F)

Q1. Predict the motion of each glider after a ‘head-on’ collision occurs, where the gliders move toward each other with the same speed.

*Student answers will vary.*

Q2. Describe the behavior of the two gliders after the collision. Was your prediction correct?

*When two gliders of the same mass are moving toward one another at the same speed and collide head-on, both gliders will move away from one another with the same speed after the collision.*

Q3. Is the collision you observed consistent with Newton’s 3<sup>rd</sup> Law? Explain.

*This result is consistent with Newton’s 3<sup>rd</sup> Law: both gliders exert an equal force on one another, and those forces impart an acceleration of the same magnitude (and opposite direction) on each glider. Because both gliders experience an acceleration of the same magnitude, each glider will move opposite the direction of their initial motion, and with equal speed.*

Q4. Calculate the momentum of each glider before and after the collision. What do you notice about the total momentum of the system before and after the collision?

*After calculating the momentum of both gliders in the instances before and after the collision between the two gliders, students should notice that the sum of the momenta before the collision is roughly equal to the sum of the momenta after the collision.*

Q5. Calculate the kinetic energy of each glider before and after the collision. What do you notice about the total kinetic energy before and after the collision? (Remember: energy is a scalar!)

*After calculating the kinetic energy of both gliders in the instances before and after the collision between the two gliders, students should notice that the sum of the kinetic energies of the two gliders is equal before and after the collision. Students should remember that kinetic energy is always a positive scalar, and that if the gliders have equal kinetic energy before and after the collision, these kinetic energies add to a net positive kinetic energy, as opposed to zero.*

Q6. Describe the behavior of the two gliders after the collision. Was your prediction correct?

*After this collision, the glider that is initially moving should come to a stop (or come close to stopping), and the stationary glider should move with the same velocity as the glider that was initially in motion.*

Q7. Is the collision you observed consistent with Newton's 3<sup>rd</sup> Law? Explain.

*Students can conclude that the two identical gliders exert equal and opposite forces on one another, as predicted by Newton's 3<sup>rd</sup> Law, since both gliders experience accelerations of the same magnitude: the glider in motion is brought to a halt by the stationary glider; the stationary glider will move with the same velocity as the glider that was initially in motion.*

Q8. Calculate the momentum of each glider before and after the collision. What do you notice about the total momentum of the system before and after the collision?

*See Q4.*

Q9. Calculate the kinetic energy of each glider before and after the collision. What do you notice about the total kinetic energy of the system before and after the collision?

*See Q5.*

Q10. Describe the behavior of the two gliders after the collision. Was your prediction correct?

*After the collision, the more massive glider and the lighter glider will move in opposite directions; the lighter glider will have a greater speed than the more massive glider.*

Q11. Is the collision you observed consistent with Newton's 3<sup>rd</sup> Law? Explain.

*This collision is also consistent with Newton's 3<sup>rd</sup> Law: while the more massive glider moves more slowly than the lighter glider, both gliders will experience an acceleration that is proportional to the force that they exert on each other and their masses.*

Q12. Calculate the momentum of each glider before and after the collision. What do you notice about the total momentum of the system before and after the collision?

*See Q4.*

Q13. Calculate the kinetic energy of each glider before and after the collision. What do you notice about the total kinetic energy of the system before and after the collision?

*See Q5.*

Q14. Describe the behavior of the two gliders after the collision. Was your prediction correct? How is this collision different from the last collision you studied?

*Instead of moving away from each other in opposite directions, both gliders will move in the direction that the more massive glider moved in before the collision. The more massive glider will move at a slower speed after the collision than it did before the collision, and the lighter glider will move faster than the more massive glider did before the collision.*

Q15. Is the collision you observed consistent with Newton's 3<sup>rd</sup> Law? Explain.

*See Q11.*

Q16. Calculate the momentum of each glider before and after the collision. What do you notice about the total momentum of the system before and after the collision?

*See Q4.*

Q17. Calculate the kinetic energy of each glider before and after the collision. What do you notice about the total kinetic energy of the system before and after the collision?

*See Q5.*

Q18. Describe the behavior of the two gliders after the collision. Was your prediction correct?

*After the collision, both gliders will still move in the same direction; however, their speeds will be different. The lighter glider will move with much greater speed than the more massive glider.*

Q19. Is the collision you observed consistent with Newton's 3<sup>rd</sup> Law? Explain.

*This collision is consistent with Newton's 3<sup>rd</sup> Law: the more massive glider experiences less acceleration than the lighter glider, and this acceleration is inversely proportional to the relative masses of both gliders.*

Q20. Calculate the momentum of each glider before and after the collision. What do you notice about the total momentum of the system before and after the collision?

*See Q4.*

Q21. Calculate the kinetic energy of each glider before and after the collision. What do you notice about the total kinetic energy of the system before and after the collision?

*See Q5.*

## **PROCEDURE G (BRIEF DESCRIPTION)**

In this experiment, students will study inelastic collisions using three gliders and the inelastic bumper (banana plug) included with this apparatus. This experiment also requires students to connect their observations to conservation of linear momentum. As in Procedure E, students can create different collisions than the ones described in the procedure.

## ANSWERS (PROCEDURE G)

Q1. Predict the motion of each glider after a ‘head-on’ collision occurs with this setup, where the gliders move toward each other with the same speed and stick together.

*Student predictions may vary.*

Q2. Describe the motion of the gliders after the collision. Was your prediction correct?

*The two gliders in this collision, moving at the same speed, should stick together and come to a stop after the collision.*

Q3. Is momentum conserved in this experiment? How can you tell?

*The momentum of the gliders is zero before the collision, since the gliders move at the same speed and both have the same mass, but move in opposite directions. The momentum after the collision is obviously zero, as the system comes to a stop after the collision. Momentum is conserved.*

Q4. Predict the motion of each glider after a ‘head-on’ collision occurs with this setup, where one glider moves toward a stationary glider, and both gliders stick together after they collide.

*Student predictions may vary.*

Q5. Describe the motion of the gliders after the collision. Was your prediction correct?

*The results of this experiment will depend on which glider is in motion before the collision. If the more massive glider is in motion before the collision, the two gliders will stick together after the collision and move with a reduced speed; if the glider with less mass is in motion, the gliders will stick together after the collision and move with lesser speed than in the other collision.*

Q6. Is momentum conserved in this experiment? How can you tell?

*The momentum before the collision is simply the product of the mass and the speed of the glider in motion; after the collision, the speed of the two-glider system is reduced as a result of the masses sticking together. Momentum is conserved.*

Q7. Predict the motion of each glider after a ‘head-on’ collision occurs with this setup, where the gliders move toward each other with the same speed and stick together.

*Student predictions may vary.*

Q8. Describe the motion of the gliders after the collision. Was your prediction correct?

*While both gliders are moving at the same speed before the collision, the gliders should stick together and move in the same direction as the more massive glider after the collision.*

Q9. Is momentum conserved in this experiment? How can you tell?

*Because both gliders are moving at the same speed, the more massive glider will have more momentum than the less massive glider. Some of the momentum of the more massive glider will appear to ‘cancel’*

*the momentum from the less massive glider, and both gliders will continue with the 'leftover' momentum. Momentum is conserved in this collision.*

### ANSWERS (ASSESSMENT G)

1a. Find the initial momentum of the system.

*See Assessment E, question 1a.*

1b. Find the final velocity of the system.

*After the two vehicles collide and stick together, they have a momentum that is given by the following equation:*

$$p_{\text{final}} = p_{\text{initial}} = m_{\text{final}}v_{\text{final}}$$

*where  $m_{\text{final}}$  is the mass of both objects. Plugging in the numbers:*

$$6750 \text{ kg m/s} = (3025 \text{ kg})v_{\text{final}}$$

$$v_{\text{final}} = 2.23 \text{ m/s.}$$

1c. Check your calculations to make sure that momentum in this system is conserved.

*See Assessment E, question 1c.*

### **PROCEDURE H (BRIEF DESCRIPTION)**

This procedure is similar to Procedure G, and also allows students to create inelastic collisions and examine them using quantitative data as well as observations. Students will notice in this experiment that while momentum is conserved in an inelastic collision, kinetic energy is not conserved.

### ANSWERS (PROCEDURE H)

Q1. Describe the motion of the gliders after the collision. Was your prediction correct?

*After the collision, the gliders should stick together and come to a stop.*

Q2. Calculate the momentum of each glider before the collision, and the momentum of the system after the collision. Is momentum conserved in this experiment? How can you tell?

*Students should calculate that the momentum of each glider is equal in magnitude and opposite in direction, and that the momentum of the system is zero after the collision. Therefore, momentum is conserved for this collision.*



Q3. Calculate the kinetic energy of each glider before the collision, and the kinetic energy of the system after the collision. Is kinetic energy conserved in this experiment?

*Each glider has a positive kinetic energy before the collision, but after the collision, the kinetic energy of each glider is zero. Kinetic energy, therefore, is not conserved in this experiment.*

Q4. Describe the motion of the gliders after the collision. Was your prediction correct?

*After the collision, the gliders stick together and move more slowly than the glider in motion moved before the collision.*

Q5. Calculate the momentum of each glider before the collision, and the momentum of the system after the collision. Is momentum conserved in this experiment? How can you tell?

*Before the collision, the glider in motion has a momentum equal to the product of its mass and its velocity. The stationary glider has a momentum of zero. Depending on student measurements, students should calculate that the system of both gliders has the same momentum as it did before the collision.*

Q6. Calculate the kinetic energy of each glider before the collision, and the kinetic energy of the system after the collision. Is kinetic energy conserved in this experiment?

*Before the collision, the only kinetic energy between the two gliders was the kinetic energy of the glider in motion. While both gliders are in motion after the collision, the system will have less kinetic energy after the collision. Kinetic energy is not conserved.*

Q7. Describe the motion of the gliders after the collision. Were your predictions correct?

*After the collision, both gliders stick together, and will move in the direction in which the more massive moved before the collision.*

Q8. Calculate the momentum of each glider before the collision, and the momentum of the system after the collision. Is momentum conserved in this experiment? How can you tell?

*Depending on student measurements, students should calculate that the momentum of the system of gliders after the collision is equal to the momentum of the system before the collision.*

Q9. Calculate the kinetic energy of each glider before the collision, and the kinetic energy of the system after the collision. Is kinetic energy conserved in this experiment?

*Depending on student measurements, students should calculate that the kinetic energy of the system of gliders after the collision is not equal to the kinetic energy of the gliders before the collision.*

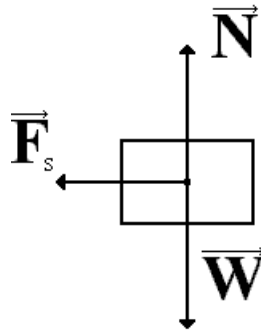
## **PROCEDURE I (BRIEF DESCRIPTION)**

This experiment allows students to observe a glider on a spring, as it undergoes simple harmonic motion. Students will examine the forces on the glider and the motion of the glider as a result of those forces. An appendix detailing a mathematical explanation of this physical situation is included in this instruction manual.

## ANSWERS (PROCEDURE I)

Q1. Keep a hold of the glider, and with your free hand, draw a free-body diagram for the glider while it is initially displaced from equilibrium. What is the direction of the net force on the glider?

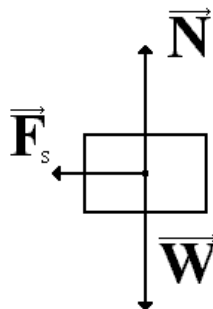
*Students should draw a free-body diagram similar to the free-body diagram below. Free-body diagrams will differ depending on how the student observes the experiment.*



*In this free-body diagram, 'N' is the normal force exerted on the glider by the air track, 'W' is the weight force exerted on the glider by the Earth, and 'F<sub>s</sub>' is the force exerted on the glider by the spring. Because the normal force and the weight force sum to zero, the net force on the glider is the force exerted on the glider by the spring. The net force on the glider, therefore, acts in the direction opposite the displacement of the spring from its equilibrium point.*

Q2. Keep a hold of the glider, and with your free hand, draw a free-body diagram for the glider at this point. Is the magnitude of the net force on the glider greater than, equal to, or less than the magnitude of the net force on the glider in Q1? How can you tell?

*Students should draw a free-body diagram similar to the free-body diagram below. Free-body diagrams will differ depending on how the student observes the experiment.*



*This free-body diagram is similar to the free-body diagram in Q1; the force exerted on the glider by the spring in this position is less than the force exerted on the glider when it is displaced 20 centimeters from equilibrium, because the spring is not displaced as far from its equilibrium point.*

Q3. Does the spring exert any force on the glider?

*If the spring is at equilibrium, it exerts no force on the glider.*

Q4. Draw a free-body diagram for the glider at this point.

*The free-body diagram for the glider when the spring is not displaced from equilibrium should resemble the free-body diagrams from Q1 and Q2, but should not include a force from the spring. However, the bumper is in contact with the end of the air track when the spring is at equilibrium, so a force exerted on the glider by the air track should be drawn in the direction opposite the forces from the spring.*

Q5. How does the net force on the glider change as it moves toward, and away from, equilibrium?

*The net force on the glider is at a maximum when the glider is furthest away from equilibrium and when the spring is at equilibrium and colliding with the air track. The net force on the glider is at a minimum when the spring is at equilibrium, just before the glider collides with the air track.*

Q6. Is the motion of the glider on the air track consistent with the free-body diagrams you drew in Q1, Q2, and Q4? What do you notice about the direction of the net force on the glider in comparison to the velocity of the glider at any given point? (Hint: think of the first part of this procedure as ‘snapshots’ of the motion of the glider.)

*The free-body diagram is an accurate reflection of the motion of the glider at a maximum point: the glider is not in motion, but will move as a result of the net force on it. The force on the glider decreases as it moves toward equilibrium. At equilibrium, the glider is still moving as a result of the force from the spring at a constant speed (for a brief instant). When the glider collides with the end of the air track, the force from the air track slows it down and sets it in motion in the opposite direction.*

Q7. How did the period of the oscillation change as the mass changed? Explain your observation in terms of the force exerted on the glider by the spring at any given time during the glider’s motion.

*The period of oscillation for the glider should increase as the mass increases. This makes sense in terms of the net force on the glider: a glider that is displaced 20 centimeters from equilibrium and has a mass of 2 kilograms will experience less acceleration than a glider displaced to the same position with a mass of 1 kilogram. Because the maximum acceleration of the glider is less, the time it takes to complete one oscillation will increase.*

## **PROCEDURE J (BRIEF DESCRIPTION)**

In this experiment, students use a glider and two springs to observe another variation of a harmonic oscillator. This experiment allows students to calculate the mass of the glider based on measurements of the period of the oscillatory motion of the glider. An appendix detailing a mathematical explanation of this physical situation is included in this instruction manual.

## **ANSWERS (PROCEDURE J)**

Q1. What is the net force on the glider at this point?

*If the glider is resting at its equilibrium point, the two springs that hold the glider in place are displaced by the same amount. Therefore, they exert equal forces on the spring which sum to zero.*

Q2. What would the net force on the glider be if you displaced it 40 centimeters from equilibrium, in either direction?

*While both springs exert a force on the glider, the spring that is displaced further from equilibrium will exert more force on the glider than the spring that is closer to equilibrium. The net force on the glider would be the difference between the two forces, and would act in the direction of the greater force.*

Q3. Using the formula for the period of this oscillating system in the Appendix, calculate the mass of the glider. Use a balance to determine the mass of the glider; how does your calculation for the mass of the glider compare the mass of the glider as determined by a balance?

*Calculations may vary. Students should find, however, that their calculations for the mass according to the period of the oscillation should match their measurements for the glider's mass to a reasonable degree of accuracy.*

Q4. Use the mass of the glider, as determined by the balance, to calculate the period of its oscillation in this system. How do the calculations compare?

*Calculations may vary. This calculation should also match measurements of the actual period of oscillation to a reasonable degree of accuracy.*

## APPENDIX TO PROCEDURES 'I' AND 'J'

### Simple Harmonic Oscillator: Glider on a Spring (Procedure 'I').

Suppose we have a glider of mass 'M' attached to a spring with spring constant 'k', resting on a frictionless surface. There is negligible effect due to air resistance. The glider is displaced from equilibrium a distance 'x' and allowed to oscillate. A diagram of this system is below. For this example, forces acting to the right are positive forces.

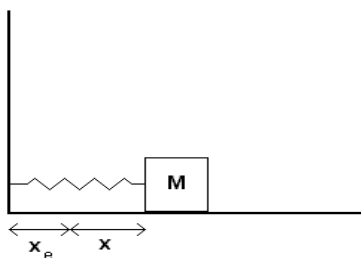


Figure 1. A glider on a spring.

The net force acting on the glider, after the weight force on the glider due to the Earth and the normal force on the glider due to the surface are summed, is simply the force due to the spring. This force can be described using Hooke's Law. The net force on the glider in this system is

$$(1) \Sigma F = -kx$$

where ' $\Sigma F$ ' is the net force on the glider, 'k' is the spring constant of the glider, and 'x' is the displacement of the spring, measured from equilibrium. There is a negative sign in the equation to show that the force due to the

spring will always act opposite the direction of the displacement. In differential equation form, this equation becomes

$$(2) M(d^2x/dt^2) = -kx$$

where ' $d^2x/dt^2$ ' is the second derivative of displacement with respect to time (acceleration). To solve the equation, all terms must be moved to one side and set equal to zero.

$$(3) M(d^2x/dt^2) + kx = 0$$

Divide by 'M', and the differential equation is in solvable form.

$$(4) (d^2x/dt^2) + (k/M)x = 0$$

To find the solution to this equation, we simply guess a function whose second derivative, when added to the function itself, is equal to zero. The best 'guess' for the function that describes 'x' is

$$(5) x = A \sin (\omega_0 t + \varphi)$$

where 'A' is the initial displacement of the oscillating glider, ' $\omega_0$ ' is the angular frequency of the oscillation of the glider, 't' is time, and ' $\varphi$ ' is the phase angle of the oscillation. Plugging in this solution, the equation becomes

$$(6) -A\omega_0^2 \sin (\omega_0 t + \varphi) = -A(k/M) \sin (\omega_0 t + \varphi)$$

When we divide the equation by 'A' and ' $\sin (\omega_0 t + \varphi)$ ', we are left with a solution for ' $\omega_0$ ':

$$(7) \omega_0^2 = k/M.$$

Take the square root of  $\omega_0$ , and the final solution comes out:

$$(8) \omega_0 = \sqrt{k/M}.$$

The angular frequency of an oscillator can be related to the period of its oscillation by the following formula:

$$(9) T = 2\pi/\omega_0,$$

where 'T' is the period of the oscillation. Replacing ' $\omega_0$ ' with its solution for this situation gives the following formula for the period of this oscillating system:

$$(10) T = 2\pi\sqrt{M/k}$$