611-1310 (40-140) Dynamics Cars

Instructions and Experiments, by Dr. P. G. Mattocks

Warranty and Parts:

We replace all defective or missing parts free of charge. All products warranted to be free from defect for 90 days**.** Does not apply to accident, misuse, or normal wear and tear.

To Assemble:

The *thinnest spring* is best for demonstrating completely inelastic collisions. Best results are obtained at mass and velocity combinations that make the *middle of the spring bend back* and almost touch the car.

The car with the spring may be turned around so that either the elastic or inelastic end collides with the other car. Thus elastic and inelastic collisions of the same masses and velocities may be compared sequentially.

To install bumpers, *unscrew knurl nuts* from front bracket of car, *bend bumper into loop*, position one hole in looped bumper over exposed screw end. Screw knurl nut back onto screw to hold bumper in place.

Materials Needed:

Weights: *(This set can hold up to **5 bricks.**) If bricks are used, we recommend sealing the brick surfaces or wrapping the bricks so paint is not scratched and bearings are not damaged by brick dust. We also recommend masking the car deck with masking tape.*

Other weights: *Lead bricks, iron bars, plastic bottles full of sand, scale weights.*

Theory:

Introduction:

When two objects collide, their subsequent motion is determined by their initial velocities and by the forces acting between the objects during the collision. There are physical constraints on those forces that lead to two conservation laws which in turn allow the subsequent motion to be predicted either fully or at least in part. The **Law of Conservation of Momentum** can be applied to all collisions regardless of the details of the collision. On the other hand, details **are** relevant in the application of the **Law of Conservation of Energy** because it is necessary to know how much energy of motion (**Kinetic Energy** - K.E.) is converted to nonrecoverable forms of energy such as heat. If that K.E. loss is known, the two conservation laws can together predict the subsequent motion.

The 40-140 Science First® Dynamics Car pair is a good method of demonstrating the relevance of momentum and energy conservation to collisions in one dimension between two objects. Each object is a car carrying a known mass. The car is supported by ball-bearing wheels. Though small, the friction in these wheels does make the cars slow down and so the observer must note the car velocities immediately **before** and **after** collision. The friction can be greatly reduced only by investing in more expensive air tracks.

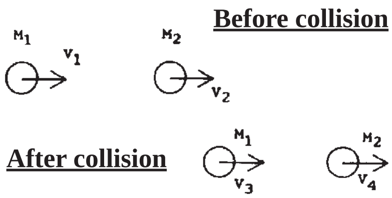
Here are special cases of mass and velocity combinations that illustrate the ideas involved without the need for timing equipment.

Conservation of Momentum:

Conservation of total momentum in a collision follows from Newton's second and third laws of motion. The third law states that **for every action there is an equal and opposite reaction.** Collisions vary in character. Duration can be short and snappy (billiard balls) or long (oil tankers colliding at sea.) The third law implies for all cases that at any instant during period of contact, the force on one object is exactly equal and opposite to the force on the other. The second law states that **the force acting on the object is numerically equal** (when appropriate units are used - newtons for force and kilogram meters per second for momentum), **to the rate of change of momentum of that object.** If the forces on the objects are equal and opposite (as required by the third law) the rates of change of momentum of those objects must be equal and opposite at every instant throughout the collision. Whatever momentum is lost by one must be gained by the other. The **total** momentum of both objects must stay the same.

Regard this conservation as an equation relating the velocities **before** (V_1 and V_2) and **after** (V_3 and V_4) the collision of Object 1 and Object 2 of Mass M_1 and M_2 respectively. The term "before" means just before the objects touch; "after" means just after the forces between the objects have dropped to zero. All velocities V are written as if the objects are moving to the right (see diagram). When numbers are inserted in the equations, a known velocity to the left would be inserted as a negative number. Similarly if the solution to an equation is a negative number, that velocity is to the left.

Do not insert a negative sign for an as yet unknown velocity even though your experience tells you it will be to the left. Use the equations to verify.



Momentum is mass times velocity, confined to one dimension.

Equation 1

$$M_1V_1 + M_2V_2 = M_1V_3 + M_2V_4$$

If the starting conditions are known, then M_1, M_2, V_1 and V_2 are known. The equation alone cannot predict V_3 and V_4 but measured values must satisfy this relation.

Conservation of Energy

Energy can be transferred from one kind to another. Some transformations are reversible, some not. Total energy is constant but its usefulness decreases through irreversible transformations.

The concept of **entropy** considers the connection between irreversibility and decreasing usefulness of energy. Irreversible energy transformations lead to decreasing usefulness equivalent to increasing disorder or increasing entropy of the universe. Reversible transformations yield no change in entropy. This is a motivation behind pleas to conserve the environment and reduce fossil fuel usage. These entropy increasing processes cannot be reversed.

Kinetic energy associated with the objects need not remain as kinetic energy of the objects. In most cases some fraction of available K.E. is irreversibly transformed to frictional heating, permanent distortion of objects, sound etc.

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Elastic Collisions

(Conservation of Kinetic Energy)

Elastic collisions are those in which **no** K.E. is irreversibly transformed to other forms of energy. Instead it is reversibly transformed to a potential energy (here, compression of a spring) and then transformed back to K.E. of the objects as they fly apart.

Perfectly elastic collisions are almost impossible to achieve. Some collisions of atomic particles in nuclear accelerators are elastic but usually we have to approximate. Two dynamics cars bouncing apart with a spring between them are an approximation in that a little energy is lost in heating the spring and in sound.

The kinetic energy of an object is

$$\mathbf{1/2MV^2}.$$

Therefore:

Equation 2

$$1/2M_1V_1^2 + 1/2M_2V_2^2 = 1/2M_1V_3^2 + 1/2M_2V_4^2$$

Equations can be combined to give V_3 and V_4 and predict exactly the subsequent motion. The expressions are given in **Appendix A.** They are not necessary, however, when some means of measuring velocity is not available.

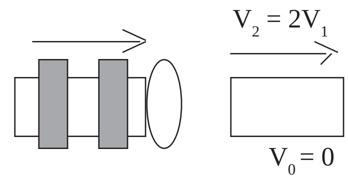
First, combine equations (see **Appendix B)** to give:

Equation 3

$$V_1 - V_2 = V_4 - V_3$$

This is an intermediate step towards the separate solutions for V_3 and V_4 in **Appendix A.** It shows the relative velocity between the objects is reversed. The objects bounce apart.

This corresponds to the **equal and opposite** forces in Newton's Third Law, having the largest effect possible. During the first part of the collision (compression of the spring) K.E. is converted to potential energy stored in the spring. Once the relative velocity has dropped to nothing the fully compressed spring pushes objects apart and ideally all energy returns to K.E.



Case 1

Elastic Collisions

(Special Cases for Demonstration)

Case 1

M_1 is much greater than M_2 gives V_3 approximately equal to V_1 and V_4 approximately equal to $2V_1 - V_2$

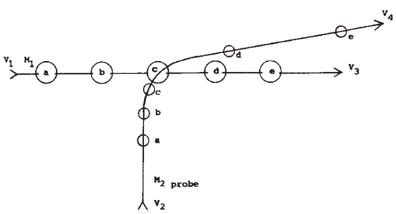
The massive object 1 is unaffected by the collision but object 2 is propelled forward (same direction as V_1) with high velocity. Demonstrate as in the diagram with 2 bricks on Car 1 **colliding with the second empty, stationary car** ($V_2 = 0$.) The empty car bounces forward at twice V_1.

Then try the same two cars **moving towards each other equally fast ($V_2 = - V_1$).** In this case, V_4 approximately equals $3V_1$ in that the light car bounces forward even faster.

If an automobile were made with a rigid front end then a head-on collision with a truck would be almost elastic and, as above, the automobile would undergo a virtually instantaneous enormous change in velocity. The occupants would be severely injured , the occupants in small cars being more so than those in large cars. Car front ends are designed to collapse, absorb energy and protect occupants in an inelastic collision.

Elastic collisions can be useful.

Consider the "gravitational slingshot" of a space probe by a planet orbiting the sun. Many space probes such as the two Voyager craft have been sent to visit many planets in the solar system sequentially. The probes had to change direction going from one planet to the next, but in order to reach the outer planets within the lifetime of the probe (and builders!) the velocity had to be high. Simply burning rocket fuel could not have achieved these two goals because of the vicious circle in which increasing the fuel carried increases the mass of the craft, therefore increasing the amount of fuel required to accelerate it.



It was realized that each encounter with a planet was an elastic collision and could be used to both change the direction and increase the probe's speed. The "collision" does not involve contact with the planet's surface, but a passage through its gravitational field. The encounter is similar to the one dimensional example demonstrated above, even though it is really a two or even three dimensional collision.

Consider the probe approaching the orbital path of a planet as sketched here. The successive positions of the probe and planet are indicated at equal intervals of time. When the planet is at **a,** the probe is at **a**, etc.

Note: *The probe passes after planet moves on. They do not physically touch.*

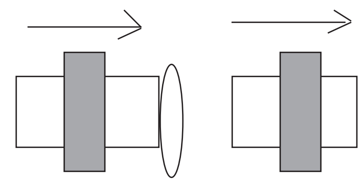
- At **a,** the separation is large, gravity is weak, the probe's path is almost straight and its speed roughly constant.
- At **b,** because the separation is smaller, gravity accelerates the probe toward the planet.
- Around **c,** the probe is strongly accelerated toward the planet, but since the planet is moving ahead, the probe will not hit it but will pass safely behind, provided the probe was aimed properly.
- Between **c** and **d,** the moving planet gives the probe a considerable increase in velocity as indicated by the increased separation between **d** and **e,** etc.
- Beyond **e** as the probe moves further from the planet, the gravitational pull that would slow it down weakens

It is not easy to see that a net increase in speed results. Gravity is like a stretched piece of elastic connecting the probe and planet, but elastic that is weak at large distances and stronger at short distances. The elastic already stores potential energy before the encounter begins. As the elastic accelerates the probe toward the planet, that potential energy is **reversibly** transformed to K.E. of the probe. Provided the probe misses the surface, the K.E. transforms back to potential energy of the re-stretched elastic. It is therefore an **elastic** encounter. There is an action and reaction pair of forces between planet and probe that obey Newton's Second and Third Laws. Therefore the encounter is a collision. Conservation of momentum and energy dictate that the lighter object is propelled forward at higher velocity. Instead of a spring exerting a push in front of the planet, gravity exerts a pull.

Imagine the extreme case where the probe comes in along **a**, **b** etc. very slowly and is timed to arrive so that its outgoing trajectory is almost parallel to the planet's path. This case is extremely close to the one dimensional collision of the cars with $V_2 = 0$. The outgoing velocity V_4 of the probe is about twice that of the planet. More realistic encounters will have a V_2 less low, the probe having been fired from Earth. The desired outgoing path will probably be outward toward another planet. Then V_4 will be less than twice V_1. But careful timing and alignment of the probe can achieve wonders. The final slingshot of one Voyager probe sent it roughly perpendicular to various planes of the planets' orbits so it could look back and photograph the Solar System from a new point of view.

Case 2

$M_1 = M_2$ gives $V_3 = V_2$ and $V_4 = V_1$

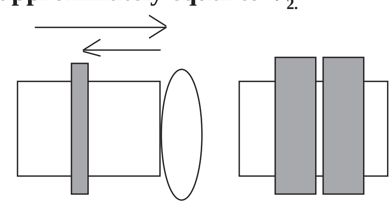


Case 2

The objects exchange velocities. This is particularly interesting when V_2 is zero. Object 1 comes to a stop and Object 2 is propelled forward. Demonstrate as in the diagram but with **equal masses** of bricks on both cars. This is the principle of the Newton's Cradle with steel balls hanging in a row. One ball swings as a pendulum, knocks the next ball on and comes to a stop. Try, too, with V_1 and V_2 **equal but opposite.**

Case 3

M_1 is greatly less than M_2 gives V_3 approximately $- V_1 + 2V_2$ and V_4 approximately equal to V_2.



Case 3

The light Object 1 is bounced back and the massive Object 2 is largely unaffected. Demonstrate as shown.

All three (3) cases are independent of spring characteristics as long as the spring is not bent too far.

Completely Inelastic Collisions:

The equations for elastic collisions indicate that the relative velocity is reversed when all of the energy stored in the spring is returned.

Suppose the spring were prevented

from rebounding. None of the stored energy would be returned. This represents the maximum possible loss of K.E. The cars would not be pushed apart.

Equation 4.

Then $\mathbf{V_3 = V_4}$.

This is the distinguishing feature of a completely inelastic collision.

The equation for momentum conservation still applies. Final velocity is:

Equation 5

$$V_3 = V_4 = M_1V_1 + M_2V_2$$

$M_1 = M_2$ gives $V_3 = V_4 = \frac{V_1 + V_2}{2}$

M_1 much less than M_2 gives

$$V_3 = V_4 = V_2$$

To illustrate this, the Dynamics Cars would need to lock together and move at roughly -the same velocity.

General Case:

Most collisions lie somewhere between the two extremes of elastic and completely inelastic. The complete range including the two extremes can be analyzed using a modification of equation:

$$\mathbf{e\,(V_1 - V_2) = V_4 - V_3}$$

where the fraction **e**, known as coefficient of restitution lies in the range 0 is less than **e** is less than 1. If e is known, from experiment or theory, equations can be solved to predict V_3 and V_4.

Collisions where e is neither 0 nor 1 are known as **inelastic** because not all K.E. is conserved.

Appendix A

For elastic collisions

$$V_3 = \frac{(M_1 - M_2)\,V_1 + 2M_2V_2}{M_1 + M_2}$$

$$V_4 = \frac{2M_1\,V_1 + (M_2 - M_1)\,V_2}{M_1 + M_2}$$

Appendix B

For elastic collisions

Equation 2 can be rearranged as:

$M_1\,(V_1^2 - V_3^2) = M_2\,(V_4^2 - V_2^2)$ OR

$M_1\,(V_1 + V_3)\,(V_1 - V_3) = M_2\,(V_4 + V_2)\,(V_4 - V_2)$

But Equation 1 becomes:

$M_1\,(V_1 - V_3) = M_2\,(V_4 - V_2)$

Divide last equation by previous:

$(V_1 + V_3) = (V_4 + V_2)$ OR

$(V_1 - V_2) = (V_4 - V_3)$

which is Equation 3.

How to Teach with Dynamic Cars

Concepts: Velocity, average & instantaneous; acceleration; uniform acceleration - relation to time, velocity & displacement; equations of motion - relation of velocity, time and distance to acceleration.

Curriculum Fit: Physics Sequence; Motion and Force. *Causes of Motion (Newton's Laws, Momentum, Energy Conservation.)* **Grades 9-10.**

Concepts: Newton's Second Law of Motion; Momentum; Conservation of Linear Momentum; Elastic, Inelastic collisions

Curriculum Fit: Motion & Force. *Momentum, a conserved quantity.* **Grades 9-10 and up.**

Concepts Taught: Conservation of Mechanical Energy.

Curriculum Fit: Physics Sequence; Energy. *Unit: Energy Conservation.* **Grades 11-12.**

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