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14015 FORCE BOARD

Purpose:

To arrange a mechanical equilibrium of vector quantities which can then be compared with graphical and various analytical methods of calculation.

Discussion:

Students have used scalar quantities (1 gallon of gasoline, 3 feet long, 90 degrees in the shade...) all of their lives and now must confront vectors. The new concept for them seems to be handling a multidimensional quantity of which "direction" is one of the dimensions. Students must come to realize that magnitude and direction must somehow be processed together in performing vector calculations or the vector won't survive. A force table in equilibrium is an ideal way to play with vector addition (since the net force on the system is zero).

Assembly:

Begin by laying the force board base disk on a table top. Place the three spring scales on the base disk. Lift the free end of the chain attached to one of the scales and pull it outward slightly from the center of the disk. Slide the nearest link of chain into one of the slots in the base disk's perimeter. Repeat this for the remaining scales.

Vector Addition, a Graphical Approach:

Vectors are quantities that come in two pieces: Magnitude (how much of the quantity there is) and Direction (what direction is the quantity going?). The magnitude by itself is a scalar (simply a number). Examples of scalars would be "5 dollars" or "2 cars". The concept that makes a vector useful is its ability to combine a scalar with a direction. Examples of vectors are: "10 kilometers northeast" or "5 meters per second downward". A vector can be drawn on a piece of paper as an arrow. The length of the arrow represents the magnitude of the vector and the direction in which the arrow points represents the vector's direction.

There are some simple general rules one needs to know about vectors before we can go further.

1) A vector can be "picked up" and moved any where you like as long as you do not change its direction or length. This simply means that you can draw the vector anywhere on the page you like.

Required Accessories:

Blank paper or graph paper
Drawing tools for graphical vector solutions
Calculator for analytical vector solutions

2) If you are adding vectors, place the "tail" of the new vector at the "head" of the old vector.

3) Subtracting a vector is the same as adding a "negative" vector.

4) To make a vector "negative" simply swap the vector's head with the vector's tail.

Now that we know how to represent vectors graphically, let's construct a few vectors on the force table and analyze them graphically.

To begin, place the ring that holds the scales together on the board's center. Evenly distribute the chains around the perimeter of the table (you don't want them all grouped on one side of the table). Make sure that none of the chains are crossed.

Balance the force board by repositioning the chains in the slots around the perimeter and by increasing or decreasing tension on the spring scales pulling the chains tighter or releasing them by moving one link of chain at a time. The force board is balanced when the ring is centered.

Once the table has been balanced, the vectors can be recorded. Each chain represents one vector. The magnitude of the vector can be recorded as the force reading on the scale. The direction of the vector can determined by counting the number of notches between chains. Each notch is separated from its neighbors by 10 degrees. Record all of your vectors.

Since the force table is balanced, the net force on the center ring is zero (it is not accelerating and there are no additional external forces acting on it beyond the three scales). This means that the sum of all the vectors on the board must equal zero. To confirm this graphically, begin by drawing your first vector in the middle of a sheet of graph paper. Pick an appropriate scale for the magnitude (length) of the vectors. Remember, once you have picked a scaling factor

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(say, one unit of force equals one centimeter on the graph paper) stick with it! Do not draw one vector with one scale
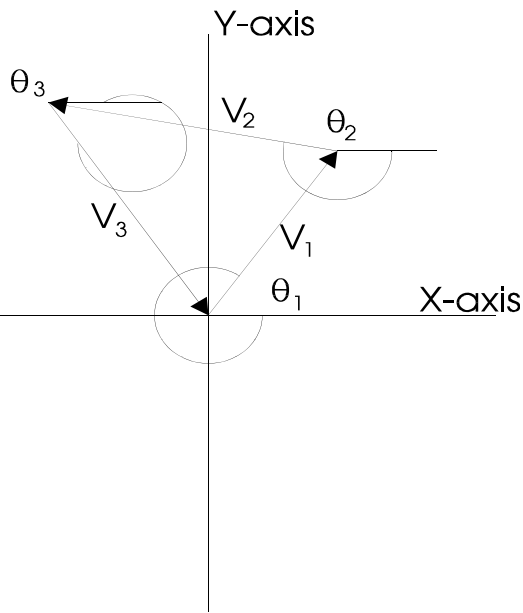
and the second vector using a different scale! Use a protractor to determine the proper angle for the vector you are drawing.

When the first vector has been drawn, continue by drawing the next vector beginning at the tip of the previous vector. Keep repeating this procedure until you run out of vectors to draw. In the ideal case (if there has been no experimental error) the tip of the last vector will end at the beginning of the first vector.

If there is some experimental error, then there will be a space between the tip of the last vector and the beginning of the first vector. Draw an arrow from the starting point of the first vector directly to the end point of the last vector. The length of this error vector represents the magnitude of the **net force** on the center ring and the angle of this vector represents the direction in which this net force is applied. Again, this last vector is a measure of your experimental error.

Vector Addition, Analytical Method:

The analytical method of adding vectors requires us to break each vector into *components*. We will do this graphically to begin with then we will use a mathematical method exclusively. As an example, we will begin by drawing an arbitrary vector on a sheet of graph paper. We can always draw a right triangle with two legs parallel to the X- and Y-axes and using the original vector as the hypotenuse. The Y-component, or the leg of the triangle parallel to the Y-axis can be determined mathematically as the magnitude of the original vector times the sine of the vector's angle counter clockwise from the positive X-axis. The X-component can be found similarly as the magnitude of the original vector times the cosine of the angle measured counter clockwise from the

positive X-axis to the original vector. Mathematically these statements are written as:

$V_X = V\cos(\theta)$
$V_Y = V\sin(\theta)$

This will give us the X and Y component of any single vector respectively. If we have several vectors, we can break them all into components then (since all of the X components are pointing in the same direction) we can add all of the X components together to get the net X component and all of the Y components together to get the net Y component. These components can be recombined into a resultant vector by applying the Pythagorean theorem to obtain the magnitude of the vector and noting that the tangent of the Y-component divided by the X-component will produce direction for the vector. In a more compact mathematical expression, this statement can be written as:

$V_{NET} = (V_{NET\,X}^2 + V_{NET\,Y}^2)^{\frac{1}{2}}$
$\theta = \arctan(V_{NET\,Y} / V_{NET\,X})$

As an example of the use of vectors, consider the following problem. Suppose an airplane flies 20.0 km in a direction 60 degrees north of east, then 30.0 km straight east, then 10.0 km straight north. How far, and in what direction is the plane from its starting point?

First, it is often helpful to sketch out the problem on graph paper. In our case we have chosen the +X axis as east, and the

+Y axis as north. This is the usual choice for maps. Let *A* represent the first displacement, *B* the second, and *C* the third. *R* represents the total displacement or the vector sum of *A*, *B*, and *C*. To find the magnitude and direction of *R*, we must first break each of our given vectors into components.

The components of *A* are:

$A_X = (20.0\ \text{km}) \cdot (\cos 60) = 10.0\ \text{km}$
$A_y = (20.0\ \text{km}) \cdot (\sin 60) = 17.3\ \text{km}$

The components of *B* are:

$B_X = (30.0 \text{ km}) \cdot (\cos 0) = 30.0 \text{ km}$
$B_y = (30.0 \text{ km}) \cdot (\sin 0) = 0.0 \text{ km}$

The components of *C* are:

$C_X = (10.0 \text{ km}) \cdot (\cos 90) = 0.0 \text{ km}$
$C_y = (10.0 \text{ km}) \cdot (\sin 90) = 10.0 \text{ km}$

All of these components can be arranged systematically in a table:

Distance	Angle (deg)	X-component	Y-component
A=20.0 km	60	10.0 km	17.3 km
B=30.0 km	0	30.0 km	0.0 km
C=10.0 km	90	0.0 km	10.0 km
		$R_X = 40.0$ km	$R_Y = 27.3$ km

The X and Y components of the resultant vector can now be combined as follows:

$R = [(40.0 \text{ km})^2 + (27.3 \text{ km})^2]^{1/2} = 48.4 \text{ km}$
$\theta = \arctan (27.3 \text{ km} / 40.0 \text{ km}) = 34.3 \text{ deg N of E}.$

Therefore, the plane traveled a distance of 48.4 km in a direction 34.3 degrees north of east.

Time Allocation:

No assembly is required on receiving this product. Individual experiment times will vary depending on methods of instruction, but normally will not exceed one class period.

Feedback:

If you have a question, a comment, or a suggestion that would improve this product, you may call our toll free number**.**